編號： 286 國立成功大學105學年度碩士班招生考試試題

系　　所：電信管理研究所

考試科目：線性代數　　考試日期：0228，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the following system of equations using the method of (a) LU decomposition and (b) Cramer's Rule. (30%)

$$\begin{aligned} x_1 + x_2 - x_3 + 2x_4 &= 7 \\ x_1 + 3x_2 + 2x_3 + 2x_4 &= 6 \\ -x_1 - 3x_2 - 4x_3 + 6x_4 &= 12 \\ 4x_2 + 7x_3 - 2x_4 &= -7 \end{aligned}$$

2. Determine the characteristic polynomial, eigenvalues, and corresponding eigenspaces of the given 4 X 4 matrix. (20%)

$$\begin{bmatrix} 4 & 2 & -2 & 2 \\ 1 & 3 & 1 & -1 \\ 0 & 0 & 2 & 0 \\ 1 & 1 & -3 & 5 \end{bmatrix}$$

3. Determine the inverse of the following 4 X 4 matrix, if it exists, using the method of Gauss-Jordan elimination. (15%)

$$\begin{bmatrix} -1 & 0 & -1 & -1 \\ -3 & -1 & 0 & -1 \\ 5 & 0 & 4 & 3 \\ 3 & 0 & 3 & 2 \end{bmatrix}$$

4. Considering the following linear programming problem,

Maximize $Z = 2x_1 + 5x_2 + 3x_3 + 4x_4 + x_5$,

subject to

$$\begin{aligned} x_1 + 3x_2 + 2x_3 + 3x_4 + x_5 &\leq 6 \\ 4x_1 + 6x_2 + 5x_3 + 7x_4 + x_5 &\leq 15 \end{aligned}$$

and

$$x_j \geq 0, \text{ for } j = 1, 2, 3, 4, 5.$$

It has been conjectured that x_1 and x_2 should be the basic variables for the optimal solution. Directly derive this basic solution (and Z) by using Gaussian elimination. (20%)

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5. Find the reduced echelon form for the following matrix. Use the reduced echelon form to determine a basis for the row space, and the rank of the matrix. (15%)

$$\begin{bmatrix} 1 & 2 & -1 & 4 \\ 0 & 1 & -2 & 3 \\ -1 & 0 & -3 & 2 \end{bmatrix}$$