國立成功大學 107 學年度碩士班招生考試試題

系　所：電信管理研究所

考試科目：線性代數

考試日期：0206，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

註：每題配分 20 %，資料或條件不足時，請自行假設。

1. Let $A = \begin{bmatrix} -8 & 0 & -20 \\ 1 & 1 & 2 \\ 4 & 0 & 10 \end{bmatrix}$，calculate A^{100}

2. Solve the following equation $\begin{cases} x + 2y - z = -1 \\ 2x + 2y + z = 1 \\ 3x + 5y - 6z = -2 \end{cases}$.

3. Solve the following system of differential equations

$$\begin{cases} x' = 3x + y + z \\ y' = 4y + 2z \\ z' = z \end{cases} \text{, where } \begin{cases} x = x(t) \\ y = y(t) \\ z = z(t) \end{cases}$$

4. Let $A = \begin{bmatrix} 2 & 2 & \cdots & 2 \\ 2 & \ddots & & \vdots \\ \vdots & & \ddots & \vdots \\ 2 & \cdots & \cdots & 2 \end{bmatrix} \in M_{n\times n}(R)$. Prove that the characteristic polynomial $f(t)$ of A is

$f(t) = (-1)^n t^{n-1}(t - 2n)$

5. Let V_1 and V_2 be vector spaces, and let $f: V_1 \to V_2$ be a linear transformation.
Let $\{\omega_1, \omega_2, \cdots, \omega_n\}$ be a linearly independent subset of $R(f)$.
Prove that if there exists a subset $K = \{v_1, v_2, \cdots, v_n\}$ of V_1
such that $f(v_i) = \omega_i$, for $i = 1, \ldots, n$, then K is also linearly independent.