編號： 268　　　　國立成功大學 108 學年度碩士班招生考試試題

系　　所：電信管理研究所

考試科目：線性代數　　　　　　　　　　　　　　　　　　　　　　　考試日期：0224，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Forward elimination changes $\mathbf{Ax}=\mathbf{b}$ to a row reduced $\mathbf{Rx}=\mathbf{d}$: the complete solution is

$$\mathbf{x}=\begin{bmatrix}4\\0\\0\end{bmatrix}+c_1\begin{bmatrix}2\\1\\0\end{bmatrix}+c_2\begin{bmatrix}5\\0\\1\end{bmatrix}$$

What is the 3 by 3 reduced row echelon matrix $\mathbf{R}$ and what is $\mathbf{d}$?

2. (20%) Find the matrix $\mathbf{P}$ that projects every vector $\mathbf{b}$ in $\mathbf{R}^3$ onto the line in the direction of $a=(2,1,3)$.

3. (20%) About the matrix $\mathbf{A}$: (a) Find the lower triangular $\mathbf{L}$ and an upper triangular $\mathbf{U}$ so that $\mathbf{A}=\mathbf{LU}$; (b) If the vector $\mathbf{b}$ is the sum of the four columns of $\mathbf{A}$, write down the complete solution to $\mathbf{Ax}=\mathbf{b}$.

$$\mathbf{A}=\begin{bmatrix}1&2&0&1\\2&4&1&4\\3&6&3&9\end{bmatrix}$$

4. (20%) Suppose that whether or not it rains today depends on previous weather conditions through the last two days. Specifically, suppose that if it has rained for the past two days, then it will rain tomorrow with probability 0.7; if it rained today but not yesterday, then it will rain tomorrow with probability 0.5; if it rained yesterday but not today, then it will rain tomorrow with probability 0.4; if it has not rained in the past two days, then it will rain tomorrow with probability 0.2. Please transform this model into a Markov chain by saying that the state at any time is determined by the weather conditions during both that day and the previous day. The process is in state 0 if it rained both today and yesterday; state 1 if it rained today but not yesterday; state 2 if it rained yesterday but not today; state 3 if it did not rain either yesterday or today. (a) Write down the transition probability matrix consisting of the four states 0, 1, 2, and 3. (b) Given that it rained on Monday and Tuesday, what is the probability that it will rain on Thursday?

5. (20%) Kevin figures that the total number of thousands of kilometers that a motorcycle can be driven before it would need to be junked is an exponential random variable with parameter $\lambda=1/20$. Henry has a used motorcycle that he claims has been driven only 10,000 kilometers. If Kevin purchases the motorcycle, what is the probability that he would get at least 20,000 additional kilometers out of it? Repeat under the assumption that the lifetime mileage of the motorcycle is not exponentially distributed but rather is (in thousands of kilometers) uniformly distributed over (0; 40).