編號： 263

國立成功大學 109 學年度碩士班招生考試試題

系　　所：電信管理研究所

考試科目：線性代數

考試日期：0211，節次：2

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Use Cramer's rule to solve

$$\begin{aligned} x_1 + 2x_2 + x_3 &= 5 \\ 2x_1 + 2x_2 + x_3 &= 6 \\ x_1 + 2x_2 + 3x_3 &= 9 \end{aligned}$$

2. (20%) If $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 3 \\ 2 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 1 \\ 5 \\ 4 \end{bmatrix}$ and $\mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{u}_2 = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$, $\mathbf{u}_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ then

$E = \{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ and $F = \{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ are ordered bases for $\mathbb{R}^3$. Let

$$x = 3\mathbf{v}_1 + 2\mathbf{v}_2 - \mathbf{v}_3 \text{ and } y = \mathbf{v}_1 - 3\mathbf{v}_2 + 2\mathbf{v}_3$$

Find the transition matrix from E to F and use it to find the coordinates of x and y with respect to the ordered basis F.

3. (20%) Let $A = \begin{bmatrix} 1 & 2 & -1 & 1 \\ 2 & 4 & -3 & 0 \\ 1 & 2 & 1 & 5 \end{bmatrix}$ and $N(A)$ denote the null space of A.

Find a basis for the row space of A and a basis for $N(A)$.

4. (20%) Suppose there are three cities A, B, and C, with the following transitions:
of the population of A, 70% remain in A; 20% move to B, and 10% move to C;
of the population of B, 25% move to A; 65% remain in B, and 10% move to C;
of the population of C, 5% move to A; 5% move to B, and 90% remain in C.
If initially there are 400,000 people in A, 200,000 in B, and 100,000 in C, then after a year, what will the population vector be?

5. (20%) Let D be the differentiation operator on P_3. Find the matrix B representing D with respect to $[1, x, x^2]$ and the matrix A representing D with respect to $[1, 2x, 4x^2 - 2]$.