

1. (a) Find the inverse of the matrix $[A]$ (10%)
 (b) Find the eigenvalues and eigenvectors of the matrix $[B]$ (10%)
 where:

$$[A] = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix} \quad [B] = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

2. Find the unit normal vector n of
 (a) plane $4x+2y+4z = -7$ (8%)
 (b) cone of revolution $z^2 = 4(x^2+y^2)$ at the point $P:(1,0,2)$ (Hint: using gradient) (12%)
3. Using least squares curve fitting to fit a straight line to the four data points
 $(-1, 1), (-1, 1.099), (0.2, 0.808), (1, 1)$
 , that is, find the values of a and b in the straight line $y=ax+b$ such that q is minimized
 where $q = \sum_{j=1}^4 (y_j - a - bx_j)^2$ (Hint: $\frac{\partial q}{\partial a} = \frac{\partial q}{\partial b} = 0$) (20%)
4. Solve the first-order differential equations
 (a) $(2x-4y+5)y' + x - 2y + 3 = 0$ (10%)
 (b) $xy' + y + 4 = 0$ (10%)
5. (a) The random variable
 $X = \text{number of heads in a single toss of a fair coin}$
 has the possible values $X=0$ and $X=1$ with probabilities $P(X=0)=1/2$ and $P(X=1)=1/2$
 Find the mean and variance of X (10%)
 (b) Describe a method that you can randomly divide ten samples into two groups. (10%)