

- (1) Let  $\{x_n\}$  be an arbitrary real sequence.
- (i) Suppose that  $\lim_{n \rightarrow \infty} x_n = r$ , what is  $\lim_{n \rightarrow \infty} |x_n| = ?$  5%
  - (ii) Conversely, if  $\lim_{n \rightarrow \infty} |x_n| = r$ , what is  $\lim_{n \rightarrow \infty} x_n = ?$  5%
- (2) Assume  $f''(x) = (x+4)e^{\frac{x-1}{2}}$ ,  $f'(1) = -10$ . Please find
- (i) where  $f$  is concave up and concave down? 5%
  - (ii) where  $f$  is increasing and decreasing? 5%
- (3) Let  $z = f(x, y) = \sqrt{16 - x^2 - y^2}$ .
- (i) Find the domain and the range of  $f$ . 5%
  - (ii) Sketch the level curves of  $f$  for  $z = 0, 1, 2, 3, 4$ . 5%
  - (iii) In what direction does  $f$  decrease most rapidly at the point  $(-1, \sqrt{3})$ ? 5%
  - (iv) Find all the critical points of  $f$ . 5%
- (4) Let  $f(x) = e^x + e^{-x} + 2 \cos x$ .
- (i) Compute the third order Taylor's expansion with remainder around  $x = 0$ . 10%
  - (ii) Use (i) to show that  $f(x)$  has a local minimum at  $x = 0$ . 10%
- (5) If  $f$  is a continuous function, find the value of the integral 10%
- $$I = \int_0^a \frac{f(x)}{f(x) + f(a-x)} dx.$$
- (Hint: let  $u = a - x$  and use the substitution method.)
- (6) Compute the double integral of  $f(x, y) = \sqrt{|y - x^2|}$  over  $S = [-1, 1] \times [0, 2]$ . 10%  
(Hint: divide  $S$  into two regions  $\{(x, y) | y < x^2\}$  and  $\{(x, y) | y \geq x^2\}$ )
- (7) The Ross-Simons Company has a monthly advertising budget of \$60,000. Their marketing department estimates that if they spend  $x$  dollars in newspaper advertising and  $y$  dollars on television advertising, then the monthly sales will be given by  $z = f(x, y) = 90x^{1/4}y^{3/4}$  dollars.
- (i) Determine how much money Ross Simons should spend on newspaper advertising and on television advertising per month in order to maximize its monthly sales. 10%
  - (ii) If the company decides to throw in additional \$2,000 as the advertising budget, use the Lagrange multiplier to estimate what the maximum monthly sales will be under the new budget plan? 10%