

20% 1. Evaluate the $\det(A)$ and the rank of A , where

$$A = \begin{bmatrix} 2 & 7 & 2 & 1 \\ -6 & 4 & 5 & 6 \\ -1 & 7 & 2 & 4 \\ -7 & 4 & 5 & 7 \end{bmatrix}$$

20% 2. Solve the ordinary differential equation of

$$(y+1)\frac{dy}{dx} + x(y^2 + 2y) = x$$

20% 3. Determine the inverse Laplace transform of

$$Y(s) = \frac{s^3 - 4s^2 + 4}{s^2(s^2 - 3s + 2)}$$

20% 4. Find the maximum value of the cost function $f = -5x_1 + x_2$ subject to the constraints

$$x_1 \geq 0, x_2 \geq 0, -x_1 + x_2 \geq -1, x_1 + x_2 \leq 6, x_2 \leq 5.$$

20% 5. A cage contains 100 mice, three of which are male. What is the probability that two male mice will be included if 10 mice are randomly selected?