

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

20% 1. Find an equation of the plane containing the line

$$x = -2 + 3t, \quad y = 4 + 2t, \quad z = 3 - t$$

and perpendicular to the plane $x - 2y + z = 5$.

20% 2. Find invertible U and V and r such that

$$UAV = \begin{bmatrix} I_r & 0 \\ 0 & 0 \end{bmatrix}, \text{ where } r = \text{rank}(A) \text{ and } A = \begin{bmatrix} 1 & 1 & 0 & -1 \\ 3 & 2 & 1 & 1 \\ 1 & 0 & 1 & 3 \end{bmatrix}.$$

20% 3. Evaluate the functions given in the following

$$(a) A = \begin{bmatrix} 1 & 0 \\ 2 & 2 \end{bmatrix} \text{ find } e^{At}; \quad (b) B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} \text{ find } \sin Bt.$$

20% 4. Gamblers A and B each roll a fair six-faced die, and B wins if his score is strictly greater than A's. What is the odds in whose favour?

20% 5. Villages A, B, C, and D are connected by overhead telephone lines joining AB, AC, BD, and CD. As a result of severe gales, there is a probability p (the same for each link) that any particular link is broken. Find the probability for a call can be made from A to B.