

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

1. Given two vector sets S_1 and S_2 with their bases below, determine whether U_1 and U_2 span the same subspace of \mathbf{R}^3 or not. (20%)

$$U_1: \{(2, 1, -1), (1, -1, 0)\}; \quad U_2: \{(1, 0, -1), (1, 2, 1), (0, 1, 1)\}$$

2. Find the determinant of matrix A . (20%)

$$A = \begin{bmatrix} 1 & 2 & 0 & 2 \\ -1 & 2 & 3 & 1 \\ -3 & 2 & -1 & 0 \\ 2 & -3 & -2 & 1 \end{bmatrix}$$

3. Let $T: \mathbf{R}^4 \rightarrow \mathbf{R}^3$ be a linear transformation defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2 - x_3 + x_4, x_1 - 2x_3 - x_4, x_1 - x_2 - 3x_3 - 3x_4)$$

Find the kernel and range of T . (20%)

4. There are three bags, each contains r red balls and b black balls. Tom picks a ball from the first bag and puts it into the second bag (without seeing the color, of course), and then picks a ball from the second bag and puts it into the third in the same way. He claims that the probability that he picks a black ball from the third bag now would be the same as if he did not move the balls at all. Do you agree? Explain your assertion. (20%)
5. Consider an equilateral triangle whose sides each have length s . Let a point be chosen uniformly from one side of the triangle. Let X denote the distance of the point chosen from the opposite vertex. Find the distribution function of X . (20%)