

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

1. Given the matrix A , find the eigenvalues and eigenvectors of A . (20%)

$$A = \begin{bmatrix} -4 & 1 & 1 & 1 \\ -16 & 3 & 4 & 4 \\ -7 & 2 & 2 & 1 \\ -11 & 1 & 3 & 4 \end{bmatrix}$$

2. Given two vector sets S_1 and S_2 with their bases below, determine whether S_1 and S_2 span the same subspace of \mathbb{R}^3 or not. (20%)

$$S_1: \{(1, 0, -1), (1, 1, 0), (0, 1, 1)\}; \quad S_2: \{(2, 1, -1), (1, 2, 1)\}$$

3. Let $T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$ be a linear transformation defined by

$$T(x_1, x_2, x_3, x_4) = (x_1 + x_2, x_3 + x_4, x_1 + x_3)$$

Find the kernel and range of T . (20%)

4. Suppose that 9 books, 2 with red covers, 3 with green covers, and 4 with blue covers, are put onto a shelf (from left to right, say). Find the probability that the books with same color are put together. (20%)

5. Let X be a continuous random variable having density f given by

$$f(x) = \frac{1}{2} e^{-|x|}, \quad -\infty < x < \infty,$$

Find $P(1 \leq |X| \leq 2)$. (20%)