

系所組別：交通管理科學系甲、乙、丙、丁組，電信管理研究所甲、乙、丙組

考試科目：統計學

考試日期：0308，節次：2

※ 考生請注意：本試題  可  不可 使用計算機

1. Which of the following statement is true? Please write out your analytical steps to show how you obtain the answer for full points (20 points)

(1) If a population distribution is skewed to the left, then given a random sample from that population, one would expect that:

- A) The mode would be less than the mean.
- B) The mode would be less than the median.
- C) The median would be less than the mean.
- D) The median would be greater than the mean.

(2) If the Z statistic (critical value) is incorrectly used in lieu of the t statistic when comparing two means from independent populations using small samples, the chance of committing a Type II error \_\_\_\_\_.

- A) Increases.
- B) Decreases
- C) Remains the same

(3) An investigator hired by a client suing for sex discrimination has developed a multiple regression model for employee salaries for the company in question. In this multiple regression model, the salaries are in thousands of dollars. For example, a data entry of 35 for the dependent variable indicates a salary of \$35,000. The indicator (dummy) variable for gender is coded as  $X_1 = 0$  if male and  $X_1 = 1$  if female. The computer output of this multiple regression model shows that the coefficient for this variable ( $X_1$ ) is -4.2. The t test showed that  $X_1$  was significant at  $\alpha = 0.1$ . This result implies that for male and female workers of the company,

- A) On the average, females earn \$4200 less.
- B) On the average, males earn \$4200 less.
- C) On the average, salaries do not differ.
- D) On the average, males have 4.2 more years of experience.
- E) On the average, females have 4.2 more years of experience.

(4) In one-way ANOVA analysis, as the between-treatment variation decreases, the probability of rejecting the null hypothesis \_\_\_\_\_.

- A) Increases.
- B) Decreases
- C) Remains the same

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2. An investor holds two stocks, each of which can rise (R), remain unchanged (U), or decline (D) on a particular day. Assuming that these stocks move independently and all outcomes are likely, please find the probability that both stocks rise; that both stocks decline; that exactly one stock declines. (10 points)

3. A high-security bank installation has installed four security systems to detect attempted break-ins. The four security systems operate independently for each other, and each has a 0.9 probability of detecting an attempt break-in. Assume an attempt break-in occurs. Please find the probability that at least one of the four security systems will detect it? (10 points)

4. A multiple regression model with four independent variables consists of 29 observations. Multiple coefficient of determination,  $R^2 = 0.8$  and the standard error,  $s = 2.0$ . Complete the analysis of variance table for this model and test the overall model for significance ( $\alpha = 0.05$ ) (10 points)

5. Let  $x_1, x_2, \dots, x_n$  be a sample of size n from a normal distribution  $N(\mu, \sigma^2)$ .

Consider the point estimator of  $\sigma^2$ :

$$\hat{\sigma}_1^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$

$$\hat{\sigma}_2^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \bar{x})^2$$

Please indicate which of these estimators are (1) Unbiased (2) Consistent? (10 points)

6. Suppose that  $X$  and  $Y$  are continuous random variables with the joint probability density function

$$f(x, y) = \begin{cases} k(x+y) & \text{for } 0 \leq x \leq 1, \quad 0 \leq y \leq 2 \\ 0 & \text{otherwise} \end{cases}$$

Please find the conditional density of  $X$  given  $Y=1/2$  and hence  $E(X|Y=1/2)$

and  $V(X|Y=1/2)$ . (10 points)

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7. Let  $\hat{u}_i$  be the residuals in the least squares fit of  $y_i$  against  $x_i$  ( $i = 1, 2, \dots, n$ ). Derive the following results: (10 points)

$$\sum_{i=1}^n \hat{u}_i = 0 \quad \text{and} \quad \sum_{i=1}^n x_i \hat{u}_i = 0$$

8. In a one-way analysis of variance with three treatments, each with five measurements, in which a completely randomized design is used, compute the F statistic where the sum of squares treatment is 17.0493 and the sum of squares error is 8.028. (10 points)

9. The following null and alternative hypotheses are given.

$$H_0 : u \leq 50$$

$$H_1 : u > 50$$

Suppose the population standard deviation is 10. The probability of a Type I error is set at 0.01 and the probability of a Type II error at 0.3. Assume that the population mean shifts from 50 to 55. How large a sample is necessary to meet these requirements? (10 points)

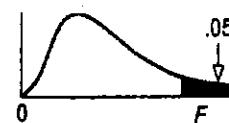
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## Critical Values of the *F* Distribution at a 5 Percent Level of Significance



|  | Degrees of Freedom for the Numerator |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |      |
|--|--------------------------------------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|------|
|  | 1                                    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   | 12   | 15   | 20   | 24   | 30   | 40   |      |
| 1                                      | 161                                  | 200  | 216  | 225  | 230  | 234  | 237  | 239  | 241  | 242  | 244  | 246  | 248  | 249  | 250  | 251  |      |
| 2                                      | 18.5                                 | 19.0 | 19.2 | 19.2 | 19.3 | 19.3 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.4 | 19.5 | 19.5 | 19.5 |      |
| 3                                      | 10.1                                 | 9.55 | 9.28 | 9.12 | 9.01 | 8.94 | 8.89 | 8.85 | 8.81 | 8.79 | 8.74 | 8.70 | 8.66 | 8.64 | 8.62 | 8.59 |      |
| 4                                      | 7.71                                 | 6.94 | 6.59 | 6.39 | 6.26 | 6.16 | 6.09 | 6.04 | 6.00 | 5.96 | 5.91 | 5.86 | 5.80 | 5.77 | 5.75 | 5.72 |      |
| 5                                      | 6.61                                 | 5.79 | 5.41 | 5.19 | 5.05 | 4.95 | 4.88 | 4.82 | 4.77 | 4.74 | 4.68 | 4.62 | 4.58 | 4.53 | 4.50 | 4.46 |      |
| 6                                      | 5.99                                 | 5.14 | 4.78 | 4.53 | 4.39 | 4.28 | 4.21 | 4.15 | 4.10 | 4.06 | 4.00 | 3.94 | 3.87 | 3.84 | 3.81 | 3.77 |      |
| 7                                      | 5.59                                 | 4.74 | 4.35 | 4.12 | 3.97 | 3.87 | 3.79 | 3.73 | 3.68 | 3.64 | 3.57 | 3.51 | 3.44 | 3.41 | 3.38 | 3.34 |      |
| 8                                      | 5.32                                 | 4.46 | 4.07 | 3.84 | 3.69 | 3.58 | 3.50 | 3.44 | 3.39 | 3.35 | 3.28 | 3.22 | 3.15 | 3.12 | 3.08 | 3.04 |      |
| 9                                      | 5.12                                 | 4.26 | 3.86 | 3.63 | 3.48 | 3.37 | 3.29 | 3.23 | 3.18 | 3.14 | 3.07 | 3.01 | 2.94 | 2.90 | 2.86 | 2.83 |      |
| 10                                     | 4.96                                 | 4.10 | 3.71 | 3.48 | 3.33 | 3.22 | 3.14 | 3.07 | 3.02 | 2.98 | 2.91 | 2.85 | 2.77 | 2.74 | 2.70 | 2.66 |      |
| Degrees of Freedom for the Denominator | 11                                   | 4.84 | 3.98 | 3.59 | 3.36 | 3.20 | 3.09 | 3.01 | 2.95 | 2.90 | 2.85 | 2.79 | 2.72 | 2.65 | 2.61 | 2.57 | 2.53 |
| 12                                     | 4.75                                 | 3.89 | 3.49 | 3.26 | 3.11 | 3.00 | 2.91 | 2.85 | 2.80 | 2.75 | 2.69 | 2.62 | 2.54 | 2.51 | 2.47 | 2.43 |      |
| 13                                     | 4.67                                 | 3.81 | 3.41 | 3.18 | 3.03 | 2.92 | 2.83 | 2.77 | 2.71 | 2.67 | 2.60 | 2.53 | 2.46 | 2.42 | 2.38 | 2.34 |      |
| 14                                     | 4.60                                 | 3.74 | 3.34 | 3.11 | 2.98 | 2.85 | 2.76 | 2.70 | 2.65 | 2.60 | 2.53 | 2.46 | 2.39 | 2.35 | 2.31 | 2.27 |      |
| 15                                     | 4.54                                 | 3.68 | 3.29 | 3.06 | 2.90 | 2.79 | 2.71 | 2.64 | 2.59 | 2.54 | 2.48 | 2.40 | 2.33 | 2.29 | 2.25 | 2.20 |      |
| 16                                     | 4.49                                 | 3.63 | 3.24 | 3.01 | 2.85 | 2.74 | 2.68 | 2.59 | 2.54 | 2.49 | 2.42 | 2.35 | 2.28 | 2.24 | 2.19 | 2.15 |      |
| 17                                     | 4.45                                 | 3.59 | 3.20 | 2.96 | 2.81 | 2.70 | 2.61 | 2.55 | 2.49 | 2.45 | 2.38 | 2.31 | 2.23 | 2.19 | 2.15 | 2.10 |      |
| 18                                     | 4.41                                 | 3.55 | 3.16 | 2.93 | 2.77 | 2.68 | 2.58 | 2.51 | 2.46 | 2.41 | 2.34 | 2.27 | 2.19 | 2.15 | 2.11 | 2.06 |      |
| 19                                     | 4.38                                 | 3.52 | 3.13 | 2.90 | 2.74 | 2.63 | 2.54 | 2.48 | 2.42 | 2.38 | 2.31 | 2.23 | 2.16 | 2.11 | 2.07 | 2.03 |      |
| 20                                     | 4.35                                 | 3.49 | 3.10 | 2.87 | 2.71 | 2.60 | 2.51 | 2.45 | 2.39 | 2.35 | 2.28 | 2.20 | 2.12 | 2.08 | 2.04 | 1.99 |      |
| 21                                     | 4.32                                 | 3.47 | 3.07 | 2.84 | 2.68 | 2.57 | 2.49 | 2.42 | 2.37 | 2.32 | 2.25 | 2.18 | 2.10 | 2.05 | 2.01 | 1.96 |      |
| 22                                     | 4.30                                 | 3.44 | 3.05 | 2.82 | 2.66 | 2.55 | 2.46 | 2.40 | 2.34 | 2.30 | 2.23 | 2.15 | 2.07 | 2.03 | 1.98 | 1.94 |      |
| 23                                     | 4.28                                 | 3.42 | 3.03 | 2.80 | 2.64 | 2.53 | 2.44 | 2.37 | 2.32 | 2.27 | 2.20 | 2.13 | 2.05 | 2.01 | 1.96 | 1.91 |      |
| 24                                     | 4.26                                 | 3.40 | 3.01 | 2.78 | 2.62 | 2.51 | 2.42 | 2.36 | 2.30 | 2.25 | 2.18 | 2.11 | 2.03 | 1.98 | 1.94 | 1.89 |      |
| 25                                     | 4.24                                 | 3.39 | 2.99 | 2.76 | 2.60 | 2.49 | 2.40 | 2.34 | 2.28 | 2.24 | 2.16 | 2.09 | 2.01 | 1.96 | 1.92 | 1.87 |      |
| 30                                     | 4.17                                 | 3.32 | 2.92 | 2.69 | 2.53 | 2.42 | 2.33 | 2.27 | 2.21 | 2.16 | 2.09 | 2.01 | 1.93 | 1.89 | 1.84 | 1.79 |      |
| 40                                     | 4.08                                 | 3.23 | 2.84 | 2.61 | 2.45 | 2.34 | 2.25 | 2.18 | 2.12 | 2.08 | 2.00 | 1.92 | 1.84 | 1.78 | 1.74 | 1.69 |      |
| 60                                     | 4.00                                 | 3.15 | 2.76 | 2.53 | 2.37 | 2.25 | 2.17 | 2.10 | 2.04 | 1.99 | 1.92 | 1.84 | 1.75 | 1.70 | 1.65 | 1.59 |      |
| 120                                    | 3.92                                 | 3.07 | 2.68 | 2.45 | 2.29 | 2.18 | 2.09 | 2.02 | 1.96 | 1.91 | 1.83 | 1.75 | 1.66 | 1.61 | 1.55 | 1.50 |      |
| $\infty$                               | 3.84                                 | 3.00 | 2.80 | 2.37 | 2.21 | 2.10 | 2.01 | 1.94 | 1.88 | 1.83 | 1.75 | 1.67 | 1.57 | 1.52 | 1.46 | 1.39 |      |

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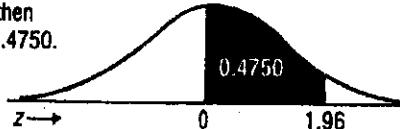
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## Areas under the Normal Curve

Example:  
If  $z = 1.96$ , then  
 $P(0 \text{ to } z) = 0.4750$ .



| $z$ | 0.00   | 0.01   | 0.02   | 0.03   | 0.04   | 0.05   | 0.06   | 0.07   | 0.08   | 0.09   |
|-----|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 0.0 | 0.0000 | 0.0040 | 0.0080 | 0.0120 | 0.0160 | 0.0199 | 0.0239 | 0.0279 | 0.0319 | 0.0359 |
| 0.1 | 0.0398 | 0.0438 | 0.0478 | 0.0517 | 0.0557 | 0.0596 | 0.0636 | 0.0675 | 0.0714 | 0.0753 |
| 0.2 | 0.0793 | 0.0832 | 0.0871 | 0.0910 | 0.0948 | 0.0987 | 0.1026 | 0.1064 | 0.1103 | 0.1141 |
| 0.3 | 0.1179 | 0.1217 | 0.1255 | 0.1293 | 0.1331 | 0.1368 | 0.1406 | 0.1443 | 0.1480 | 0.1517 |
| 0.4 | 0.1554 | 0.1591 | 0.1628 | 0.1664 | 0.1700 | 0.1736 | 0.1772 | 0.1808 | 0.1844 | 0.1879 |
| 0.5 | 0.1915 | 0.1950 | 0.1985 | 0.2019 | 0.2054 | 0.2088 | 0.2123 | 0.2157 | 0.2190 | 0.2224 |
| 0.6 | 0.2257 | 0.2291 | 0.2324 | 0.2357 | 0.2389 | 0.2422 | 0.2454 | 0.2486 | 0.2517 | 0.2549 |
| 0.7 | 0.2580 | 0.2611 | 0.2642 | 0.2673 | 0.2704 | 0.2734 | 0.2764 | 0.2794 | 0.2823 | 0.2852 |
| 0.8 | 0.2881 | 0.2910 | 0.2939 | 0.2967 | 0.2995 | 0.3023 | 0.3051 | 0.3078 | 0.3106 | 0.3133 |
| 0.9 | 0.3159 | 0.3186 | 0.3212 | 0.3238 | 0.3264 | 0.3289 | 0.3315 | 0.3340 | 0.3365 | 0.3389 |
| 1.0 | 0.3413 | 0.3438 | 0.3461 | 0.3485 | 0.3508 | 0.3531 | 0.3554 | 0.3577 | 0.3599 | 0.3621 |
| 1.1 | 0.3643 | 0.3665 | 0.3686 | 0.3708 | 0.3729 | 0.3749 | 0.3770 | 0.3790 | 0.3810 | 0.3830 |
| 1.2 | 0.3849 | 0.3869 | 0.3888 | 0.3907 | 0.3925 | 0.3944 | 0.3962 | 0.3980 | 0.3997 | 0.4015 |
| 1.3 | 0.4032 | 0.4049 | 0.4066 | 0.4082 | 0.4099 | 0.4115 | 0.4131 | 0.4147 | 0.4162 | 0.4177 |
| 1.4 | 0.4192 | 0.4207 | 0.4222 | 0.4236 | 0.4251 | 0.4265 | 0.4279 | 0.4292 | 0.4306 | 0.4319 |
| 1.5 | 0.4332 | 0.4345 | 0.4357 | 0.4370 | 0.4382 | 0.4394 | 0.4406 | 0.4418 | 0.4429 | 0.4441 |
| 1.6 | 0.4452 | 0.4463 | 0.4474 | 0.4484 | 0.4495 | 0.4505 | 0.4515 | 0.4525 | 0.4535 | 0.4545 |
| 1.7 | 0.4554 | 0.4564 | 0.4573 | 0.4582 | 0.4591 | 0.4599 | 0.4608 | 0.4616 | 0.4625 | 0.4633 |
| 1.8 | 0.4641 | 0.4649 | 0.4656 | 0.4664 | 0.4671 | 0.4678 | 0.4686 | 0.4693 | 0.4699 | 0.4706 |
| 1.9 | 0.4713 | 0.4719 | 0.4726 | 0.4732 | 0.4738 | 0.4744 | 0.4750 | 0.4756 | 0.4761 | 0.4767 |
| 2.0 | 0.4772 | 0.4778 | 0.4783 | 0.4788 | 0.4793 | 0.4798 | 0.4803 | 0.4808 | 0.4812 | 0.4817 |
| 2.1 | 0.4821 | 0.4826 | 0.4830 | 0.4834 | 0.4838 | 0.4842 | 0.4846 | 0.4850 | 0.4854 | 0.4857 |
| 2.2 | 0.4861 | 0.4864 | 0.4868 | 0.4871 | 0.4875 | 0.4878 | 0.4881 | 0.4884 | 0.4887 | 0.4890 |
| 2.3 | 0.4893 | 0.4896 | 0.4898 | 0.4901 | 0.4904 | 0.4906 | 0.4909 | 0.4911 | 0.4913 | 0.4916 |
| 2.4 | 0.4918 | 0.4920 | 0.4922 | 0.4925 | 0.4927 | 0.4929 | 0.4931 | 0.4932 | 0.4934 | 0.4936 |
| 2.5 | 0.4938 | 0.4940 | 0.4941 | 0.4943 | 0.4945 | 0.4946 | 0.4948 | 0.4949 | 0.4951 | 0.4952 |
| 2.6 | 0.4953 | 0.4955 | 0.4956 | 0.4957 | 0.4959 | 0.4960 | 0.4961 | 0.4962 | 0.4963 | 0.4964 |
| 2.7 | 0.4965 | 0.4966 | 0.4967 | 0.4968 | 0.4969 | 0.4970 | 0.4971 | 0.4972 | 0.4973 | 0.4974 |
| 2.8 | 0.4974 | 0.4975 | 0.4976 | 0.4977 | 0.4977 | 0.4978 | 0.4979 | 0.4979 | 0.4980 | 0.4981 |
| 2.9 | 0.4981 | 0.4982 | 0.4982 | 0.4983 | 0.4984 | 0.4984 | 0.4985 | 0.4985 | 0.4986 | 0.4986 |
| 3.0 | 0.4987 | 0.4987 | 0.4987 | 0.4988 | 0.4988 | 0.4989 | 0.4989 | 0.4989 | 0.4990 | 0.4990 |