※ 考生請注意：本試題 □可 ☑不可 使用計算機

(1) (20%) Find the eigenvalues of $A=\begin{bmatrix}10 & 1 & -7 & -8 & -9\\ 0 & 5 & 0 & 9 & 8\\ 6 & -4 & -3 & 6 & 4\\ 0 & 0 & 0 & -4 & -6\\ 0 & 0 & 0 & 3 & 5\end{bmatrix}$.

(2) (20%) Let T be a linear transformation and

$T(a+bx+cx^2)=(a-b-2c)+(2a-3b-5c)x+(-a+3b+5c)x^2$.

Find $T^{-1}(a+bx+cx^2)=?$

(3) (20%) Let X be a 2×2 matrix.

$X^2-5X+3I_2=\begin{bmatrix}-5 & 2\\ -4 & 1\end{bmatrix}$, where I_2 is the 2×2 identity matrix.

Find $X=?$

(4) (20%) In a poker hand what is the probability of getting exactly two pairs? Here, a hand such as $(2,2,2,2,x)$ does not count as two pairs but as a 4-of-a-kind.

(5) (20%) We assume that the number of earthquakes occurring in Taiwan in any interval of length t is a Poisson random variable with parameter λt. Suppose that earthquakes occur in Taiwan at a rate of 2 per week.

(a) Find the probability that at least 3 earthquakes occurring during the next 2 weeks.

(b) Find the probability distribution of the time, starting from now, until the next earthquake.