※ 考生請注意:本試題 □可 ☑不可 使用計算機

1. Find the answer of $Q = \int_0^1 \int_0^{x_n} \cdots \int_0^{x_3} \int_0^{x_2} dx_1 dx_2 \cdots dx_{n-1} dx_n$. (5 %)

2. Find the Taylor polynomial of order n about 0 for $f(x) = (1+x)^n$, $n > 0$ is an integer, and $|x| < 1$. (5%)

3.

   (a) Let $b_n = \dfrac{1}{n^2}$. Is $\sum b_n$ convergent or divergent? (5%)

   (b) Let $a_n = \dfrac{4n}{5n^3 - 1}$, $n \geq 1$. Is $\sum a_n$ convergent or divergent? (5%)

4. Evaluate: (20%, 4% each)

   (a) $\int (x+2)\sin(x^2 + 4x - 6)dx$. (b) $\int_{-1}^1 \dfrac{dx}{\sqrt{(x+2)(3-x)}}$. (c) $\int \dfrac{x\,dx}{\sqrt{x^2 + x + 1}}$.

   (d) $\int_0^1 |2x^2 - x| dx$. (e) $\int_0^\infty x^6 e^{-2x} dx$.

5.

   (a) If $x^2 y + y^3 = 2$, find (i) y', (ii) y'' at the point $(1,1)$. (8%)

   (b) If $y''(t) + (2+t)y'(t) + (2+t^2)y(t) = 0$, $y(0) = 2$, and $y'(0) = 0$, find (i) $y''(0)$ and (ii) $y'''(0)$. (10%)

6. The GDP of Country A is given by $Y(K,L) = 80K^{0.2}L^{0.8}$ in which K is the quantity of capital and L is the quantity of labor employed. The cost of labor per unit is \$20 while the cost of capital is \$40 per unit. The sum of cost of capital and labor is constrained to be \$600,000. What are the numbers of units of capital and labor that maximize the GDP of Country A. (12%)

(背面仍有題目,請繼續作答)

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7. The population sizes of eagles (E) and turkeys (C) at time t are described as the following differential equations: (E and C both are measured in thousands while t is measured in years)

$$\frac{dE}{dt} = 0.09(C-7),$$
$$\frac{dC}{dt} = -0.16(E-3)$$

Suppose that the initial population of eagles is 3,600 and the initial population of turkeys is 9,000. Find the population functions of E and C. (16%)

8. The following figure demonstrates the differential equation (autonomous equation) $\frac{dy}{dx} = f(y)$
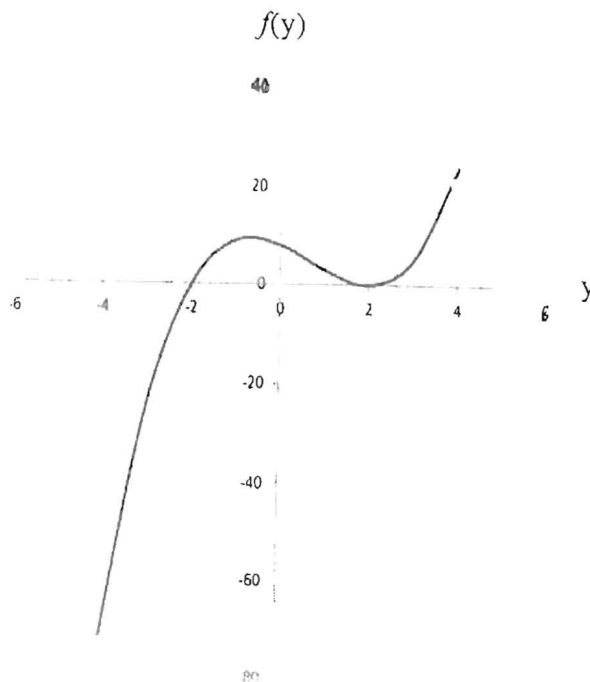
(a) Classify each equilibrium solution of the autonomous equation as sink, node or source. (4%)
(b) The possible function of $f(y)$. (5%)
(c) The rough drawing of direction fields (slope fields) that correspond to the autonomous equation $\frac{dy}{dx} = f(y)$. (5%)