國立成功大學 84 學年度 國際企業所（乙組）考試（微積分 試題） 共 1 頁 第 1 頁

1. Set
$$g(x,y)=\begin{cases}\dfrac{x^2y^2}{x^4+y^4}, & (x,y)\neq(0,0)\\ 0, & (x,y)=0\end{cases}$$

(a) Show that $\frac{\partial g}{\partial x}$ and $\frac{\partial g}{\partial y}$ both exist at $(0,0)$. What are their values at $(0,0)$?

(b) Show that $\lim_{(x,y)\to(0,0)} g(x,y)$ does not exist. (15%)

2. Find the volume of the solid bounded above by $z=x^3y$ and below by the triangle with vertices $(0,0)$, $(2,0)$, $(0,1)$. (15%)

3. A manufacturer makes 3 products: A, B, and C on which the profit per unit is \$6, \$8, and \$10, respectively. The daily levels of production are x, y, and z, respectively. The research department has decided that x, y, and z must be subject to the constraint: $x^2+y^2+z^2=800$. Find the levels of production of each product that will maximize daily profit. (15%)

4. A telephone pole is 15 ft away from a street light; the latter is 20 ft above the ground. A squirrel runs up the telephone pole at 8 ft/sec. How fast is the squirrel's shadow traveling along the (level) ground when the squirrel is 18 ft above the ground? (15%)

5. Evaluate (a) $\int_0^1 \sqrt{x^4+x^7}\,dx$ (b) $\int \frac{1}{1+\cos 2x}\,dx$ (10%)

6. The region R in the first quadrant is bounded above by the graph of $y=\sqrt{\sin x}$ and below by the x-axis, $0\le x\le\pi$. When it is rotated around the x-axis it generates a solid of volume V. Find V. (10%)

7. If Σa_n and Σb_n are both convergent series, is $\Sigma a_n b_n$ convergent? Explain. (10%)

8. Find the area bounded by one loop of the graph of the polar equation $r=\sin\theta\cos\theta$ (10%)