

注意:未寫明演算過程者不予計分

1. Find the following values, if exist.

(30%)

(a)  $\int_{-1}^5 |x^2 - 3| dx$

(b)  $\lim_{n \rightarrow \infty} n(a^{1/n} - 1)$

(c)  $\lim_{x \rightarrow \infty} (1 - \frac{1}{x^2})^x$

(d)  $\int_0^{\infty} \frac{\sin u}{u} du$

(e)  $\int_0^{\pi} e^{2x} \cos x dx$

(f)  $\int_1^e \sin(\ln x) dx$

2. Determine whether the given series converges or diverges.

(10%)

(a)  $\sum_{k=1}^{\infty} \frac{k!}{k^k}$

(b)  $\sum_{k=1}^{\infty} \sin(\frac{\pi}{k})$

3. Find  $\frac{dy}{dx}$  for the following  $y$ .

(10%)

(a)  $y = \frac{\sqrt{1+x^2}(3x+2)^3}{\sqrt[3]{x^2(x+1)}}$

(b)  $y = \int_{\ln x}^{x^2} \cos^3 t dt$

4. Prove that all normals to the curve  $x^2 + y^2 = a^2$  pass through the origin.

(10%)

5. Find the area bounded by the graphs of the equations  $3y - x = 6$ ,  $x + y = -2$  and  $x + y^2 = 4$ . (10%)

6. Find the length of the graph of  $y = \frac{1}{2}(e^x + e^{-x})$  from  $(0, 1)$  to  $(\ln 2, 5/4)$ .

(10%)

7. Is the function

$$f(x, y) = \begin{cases} \frac{x^2}{x^2+y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

continuous at  $(0, 0)$ ?

(10%)

8. Let  $\Omega$  be the region bounded by  $x = 1$ ,  $y = 0$ , and  $y = x$ . Evaluate  $\iint_{\Omega} \frac{1}{(1+x^2+y^2)^{3/2}} dx dy$ . (10%)