1. 若某公司有幾萬名員工，分成生產、行銷、研發 3 個部門，公司經理想知道員工對公司某項政策是否贊成，做抽樣調查，各部門贊成與否人數統計如下

| | 生產 | 行銷 | 研發 | |
|---|---|---|---|---|
| 贊成 | 80 | 60 | 30 | 170 |
| 不贊成 | 70 | 40 | 20 | 130 |
| | 150 | 100 | 50 | 300 |

請問(顯著水準$\alpha=0.05$) (15%)

(1) 是否有證據說公司員工贊成此項政策的比例超過 50%？

(2) 是否有證據說生產部門贊成此項政策的比例少於其他部門？

(3) 是否有證據說公司 3 個部門贊成此政策的比例不一致？

2. 若某百貨公司欲了解男女顧客的平均消費金額是否有顯著差異，隨機抽樣 100 位男生，150 位女生，結果男生平均每次消費 1800 元，標準差 500 元，女生平均每次消費 1500 元，標準差 450 元，(顯著水準$\alpha=0.05$)

(1) 請問男女生每次消費金額之標準差是否有顯著差異？ (15%)

(2) 請問男女生每次消費金額之平均數是否有顯著差異？

(3) 求性別與每次消費金額之相關係數有多高？

3. 若某種潛能訓練課程聲稱可以提高人的潛能，有 100 位學員參加此課程，在訓練前測得平均潛能是 65 分，標準差是 10 分，訓練後平均潛能是 70 分，標準差是 8 分，並測得訓練後與訓練前潛能相差平均是 5 分，標準差是 9 分，(顯著水準$\alpha=0.05$) (15%)

(1) 請問此訓練課程是否有效？

(2) 請問訓練前與訓練後潛能的相關性有多高？

(3) 請寫出訓練後潛能對訓練前潛能的迴歸式？

4. $y$ 對 $x_1$、$x_2$ 複迴歸，模式為

$$y_i=\beta_0+\beta_1 x_{1i}+\beta_2 x_{2i}+\varepsilon_i$$
$$i=1,\ldots,n$$

參數估計表與 ANOVA 表如下



(背面仍有題目,請繼續作答)

| | B | ST. ERR. | T_17_ | P_LEVEL |
|---|---|---|---|---|
| Intercept | -.09769 | 19.18749 | -.005091 | .995997 |
| X1 | 45.60207 | 75.42921 | .604568 | .553450 |
| X2 | 12.25694 | 2.05423 | 6.453475 | .000006 |

| | SUMS OF | DF | MEAN_SQU | F | P_LEVEL |
|---|---|---|---|---|---|
| Regress. | 2966.284 | 2 | 1483.142 | 32.50291 | .000002 |
| Residual | 775.728 | 17 | 45.631 | | |
| Total | 3742.012 | 19 | | | |

(1) 樣本數有多大？ (15%)

(2) 在$\alpha = 0.05$下，檢定$H_0$: $\beta_1 = \beta_2 = 0$是否顯著？為什麼？

(3) $x_1$、$x_2$對$y$解釋變異的比例有多少？

(4) $x_1$、$x_2$對$y$的影響，何者較大？為什麼？

(5) 您會建議用$y$對$x_1$、$x_2$的複迴歸或是$y$對$x_1$的簡單迴歸，或是$y$對$x_2$的簡單迴歸？請說出理由。

5. 某人做一個實驗設計，影響因素有 A、B 兩個，因素 A 有 2 個水準，因素 B 有 3 個水準，6 個配方都各做 20 次重覆實驗。模式為

$$y_{ijk} = \mu + \alpha_i + \beta_j + (\alpha\beta)_{ij} + \varepsilon_{ijk}$$

$$i = 1, 2，j = 1, 2, 3，k = 1, \ldots, 20$$

算出部份主效用與交互作用的估計及 120 個資料的標準差 $S$ 如下

$\hat{\alpha}_1 = 2$，$\hat{\beta}_1 = -1$，$\hat{\beta}_2 = -2$，$(\widehat{\alpha\beta})_{11} = 1$，$(\widehat{\alpha\beta})_{12} = -2$，$S = 8$ (25%)

(1) 求$\hat{\alpha}_2$，$\hat{\beta}_3$，$(\widehat{\alpha\beta})_{21}$，$(\widehat{\alpha\beta})_{22}$，$(\widehat{\alpha\beta})_{23}$

(2) 求 $SSA$、$SSE$

(3) 檢定是否有交互作用？($\alpha = 0.05$)

(4) 檢定是否有主效用？($\alpha = 0.05$)

(5) 寫出 ANOVA 表

6. 甲、乙兩組母體都是常態分配，且標準差都等於 4，但甲母體比乙母體的平均數多 4，試問 (15%)

(1) 從甲、乙兩母體各隨機抽 1 個樣本，抽到甲的樣本比乙的樣本大的機會是多少？

(2) 從甲、乙兩母體各隨機抽 8 個樣本，抽到甲的樣本平均數比乙的樣本平均數大的機會是多少？

(3) 每次從甲、乙兩母體各隨機抽 1 個樣本，共抽 8 次，這 8 次數據都是甲比乙大的機會是多少？

附表

$F_{1,120,0.05}=3.9201$,　$F_{2,120,0.05}=3.0718$,　$F_{1,120,0.025}=5.1523$,　$F_{2,120,0.025}=3.8046$

$F_{1,100,0.05}=3.9361$,　$F_{2,100,0.05}=3.0873$,　$F_{1,100,0.025}=5.1786$,　$F_{2,100,0.025}=3.8284$

$F_{90,150,0.05}=1.3560$,　$F_{100,150,0.05}=1.3448$,　$F_{90,150,0.025}=1.4374$,　$F_{100,150,0.025}=1.4234$

$F_{90,100,0.05}=1.4020$,　$F_{100,100,0.05}=1.3917$,　$F_{90,100,0.025}=1.4963$,　$F_{100,100,0.025}=1.4833$

$\chi^2_{2,0.05}=5.9915$,　$\chi^2_{2,0.025}=7.3778$,　$\chi^2_{3,0.05}=7.8147$,　$\chi^3_{3,0.025}=9.3484$

常態分配表

| | z的小數點第二位 | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| z | 0.00 | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 | 0.06 | 0.07 | 0.08 | 0.09 |
| 0.0 | 0.5000 | 0.5040 | 0.5080 | 0.5120 | 0.5160 | 0.5199 | 0.5239 | 0.5279 | 0.5319 | 0.5359 |
| 0.1 | 0.5398 | 0.5438 | 0.5478 | 0.5517 | 0.5557 | 0.5596 | 0.5636 | 0.5675 | 0.5714 | 0.5753 |
| 0.2 | 0.5793 | 0.5832 | 0.5871 | 0.5910 | 0.5948 | 0.5987 | 0.6026 | 0.6064 | 0.6103 | 0.6141 |
| 0.3 | 0.6179 | 0.6217 | 0.6255 | 0.6293 | 0.6331 | 0.6368 | 0.6406 | 0.6443 | 0.6480 | 0.6517 |
| 0.4 | 0.6554 | 0.6591 | 0.6628 | 0.6664 | 0.6700 | 0.6736 | 0.6772 | 0.6808 | 0.6844 | 0.6879 |
| 0.5 | 0.6915 | 0.6950 | 0.6985 | 0.7019 | 0.7054 | 0.7088 | 0.7123 | 0.7157 | 0.7190 | 0.7224 |
| 0.6 | 0.7257 | 0.7291 | 0.7324 | 0.7357 | 0.7389 | 0.7422 | 0.7454 | 0.7486 | 0.7517 | 0.7549 |
| 0.7 | 0.7580 | 0.7611 | 0.7642 | 0.7673 | 0.7704 | 0.7734 | 0.7764 | 0.7794 | 0.7823 | 0.7852 |
| 0.8 | 0.7881 | 0.7910 | 0.7939 | 0.7967 | 0.7995 | 0.8023 | 0.8051 | 0.8078 | 0.8106 | 0.8133 |
| 0.9 | 0.8159 | 0.8186 | 0.8212 | 0.8238 | 0.8264 | 0.8289 | 0.8315 | 0.8340 | 0.8365 | 0.8389 |
| 1.0 | 0.8413 | 0.8438 | 0.8461 | 0.8485 | 0.8508 | 0.8531 | 0.8554 | 0.8577 | 0.8599 | 0.8621 |
| 1.1 | 0.8643 | 0.8665 | 0.8686 | 0.8708 | 0.8729 | 0.8749 | 0.8770 | 0.8790 | 0.8810 | 0.8830 |
| 1.2 | 0.8849 | 0.8869 | 0.8888 | 0.8907 | 0.8925 | 0.8944 | 0.8962 | 0.8980 | 0.8997 | 0.9015 |
| 1.3 | 0.9032 | 0.9049 | 0.9066 | 0.9082 | 0.9099 | 0.9115 | 0.9131 | 0.9147 | 0.9162 | 0.9177 |
| 1.4 | 0.9192 | 0.9207 | 0.9222 | 0.9236 | 0.9251 | 0.9265 | 0.9279 | 0.9292 | 0.9306 | 0.9319 |
| 1.5 | 0.9332 | 0.9345 | 0.9357 | 0.9370 | 0.9382 | 0.9394 | 0.9406 | 0.9418 | 0.9429 | 0.9441 |
| 1.6 | 0.9452 | 0.9463 | 0.9474 | 0.9484 | 0.9495 | 0.9505 | 0.9515 | 0.9525 | 0.9535 | 0.9545 |
| 1.7 | 0.9554 | 0.9564 | 0.9573 | 0.9582 | 0.9591 | 0.9599 | 0.9608 | 0.9616 | 0.9625 | 0.9633 |
| 1.8 | 0.9641 | 0.9649 | 0.9656 | 0.9664 | 0.9671 | 0.9678 | 0.9686 | 0.9693 | 0.9699 | 0.9706 |
| 1.9 | 0.9713 | 0.9719 | 0.9726 | 0.9732 | 0.9738 | 0.9744 | 0.9750 | 0.9756 | 0.9761 | 0.9767 |
| 2.0 | 0.9772 | 0.9778 | 0.9783 | 0.9788 | 0.9793 | 0.9798 | 0.9803 | 0.9808 | 0.9812 | 0.9817 |
| 2.1 | 0.9821 | 0.9826 | 0.9830 | 0.9834 | 0.9838 | 0.9842 | 0.9846 | 0.9850 | 0.9854 | 0.9857 |
| 2.2 | 0.9861 | 0.9864 | 0.9868 | 0.9871 | 0.9875 | 0.9878 | 0.9881 | 0.9884 | 0.9887 | 0.9890 |
| 2.3 | 0.9893 | 0.9896 | 0.9898 | 0.9901 | 0.9904 | 0.9906 | 0.9909 | 0.9911 | 0.9913 | 0.9916 |
| 2.4 | 0.9918 | 0.9920 | 0.9922 | 0.9925 | 0.9927 | 0.9929 | 0.9931 | 0.9932 | 0.9934 | 0.9936 |
| 2.5 | 0.9938 | 0.9940 | 0.9941 | 0.9943 | 0.9945 | 0.9946 | 0.9948 | 0.9949 | 0.9951 | 0.9952 |
| 2.6 | 0.9953 | 0.9955 | 0.9956 | 0.9957 | 0.9959 | 0.9960 | 0.9961 | 0.9962 | 0.9963 | 0.9964 |
| 2.7 | 0.9965 | 0.9966 | 0.9967 | 0.9968 | 0.9969 | 0.9970 | 0.9971 | 0.9972 | 0.9973 | 0.9974 |
| 2.8 | 0.9974 | 0.9975 | 0.9976 | 0.9977 | 0.9977 | 0.9978 | 0.9979 | 0.9979 | 0.9980 | 0.9981 |
| 2.9 | 0.9981 | 0.9982 | 0.9982 | 0.9983 | 0.9984 | 0.9984 | 0.9985 | 0.9985 | 0.9986 | 0.9986 |
| 3.0 | 0.9987 | 0.9987 | 0.9987 | 0.9988 | 0.9988 | 0.9989 | 0.9989 | 0.9989 | 0.9990 | 0.9990 |
| 3.1 | 0.9990 | 0.9991 | 0.9991 | 0.9991 | 0.9992 | 0.9992 | 0.9992 | 0.9992 | 0.9993 | 0.9993 |
| 3.2 | 0.9993 | 0.9993 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9994 | 0.9995 | 0.9995 | 0.9995 |
| 3.3 | 0.9995 | 0.9995 | 0.9995 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9996 | 0.9997 |
| 3.4 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9997 | 0.9998 |
| 3.5 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 | 0.9998 |
| 3.6 | 0.9998 | 0.9998 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.7 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.8 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 | 0.9999 |
| 3.9 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |
| 4.0 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 | 1.0000 |

10. Consider the following statements regarding matrix operations:

I. The squareness is a sufficient condition for the existence of an inverse of matrix **A**.

II. For matrixes **A** and **B**, if **AB=0**, then the only conclusions are ① either **A=0** or **B=0**, or ② both **A** and **B** are zeros.

III. For matrixes **C**, **D**, and **E**, if **CD=CE**, **C**$\neq$**0**, then **D=E**.

Which of the above three statements is (or are) true?

(A) I and II, (B) II and III, (C) I and III, (D) I, II, and III, (E) none of them.

11. Let $(x+1)\dfrac{dy}{dx}=2y$. The solution to this differential equation is

(A) y=ln(1+x) + c, (B) y = x+1+c, (C) y=(x+1)$^2$ + c, (D) y=c(x+1)$^2$, (E) $y=ce^{(x+1)}$

12. A tennis club will charge an annual membership fee of \$200 per member if 100 members or less joint. For each additional member, the fee is reduced by 50 cents. The club will admit at most 300 members. What is the size of membership that maximizes revenue for the tennis club?

(A) 300, (B) 250, (C) 200, (D) 150, (E) 100.

貳、應用題: 共 40 分。

1. Assume that the utility function of a person for hamburgers (Y) and soft drinks (X) is $U(X,Y)=\sqrt{XY}$. Assume that hamburgers cost \$1 each, soft drinks cost \$0.25, and that this person has \$2 to spend.

(A) Find the best combination of hamburgers and soft drinks which will maximizes his personal utility. (4 分)

(B) Please check the second order condition for maximization. (3 分)

(C) How much utility will be raised for an increase in income of \$1? (3 分)

2. The moment generating function of a random variable X is defined as $M(t)=E(e^{tX})$, where E is the operator of expectation. Using the fact that the *m*th moment of distribution of the random variable $E(X^m)=M^{(m)}(0)$, calculate the mean ($\mu$) and variance ($\sigma^2$) of the following gamma distribution

$$f(x)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}x^{\alpha-1}e^{-x/\beta},\quad 0<x<\infty$$ (15 分)

$$=0,\quad \text{elsewhere}$$

3. (A) Prove that $\int_0^{\infty} e^{-x^2}dx=\dfrac{\sqrt{\pi}}{2}$ (10 分)

(B) Evaluate $\int_0^{\infty}\dfrac{1}{\sqrt{2\pi}}e^{-x^2/2}dx$ (5 分)