(20%) (一) Which of the following statements are true and which is false?

(1) If $f'(c)=0$, then $f(x)$ has a maximum or minimum value at $x=c$.

(2) If $f'(x)=g'(x)$ for all x in an interval I, then $f(x)=g(x)$ on I.

(3) If $f(x)$ is differentiable on the open interval (a,b), and c is a point of local maximum for f in (a,b), then $f'(c)=0$.

(4) If $f''(a)$ exists, then f' is continuous at a.

(5) If $f''(xo)=0$, then $(xo, f(xo))$ is an inflection point.

(6) If $\int_a^b f(x)dx=\int_a^b g(x)dx$, then $f(x)=g(x)$.

(7) If $\lim_{n\to\infty} a_n=0$, then $\sum a_n$ converges.

(8) If $\sum_{n=1}^{\infty} a_n$ converges, then $\lim a_n=0$.

(9) If $\sum_{n=1}^{\infty} a_n$ is a series of nonnegative terms, and $a_1+a_2+\cdots+a_n\le 5$ for all n, then $\sum a_n$ converges.

(10) If $\sum_{n=1}^{\infty} a_n$ is a series of positive terms and $a_n+1/a_n<1$ for all n, then $\sum_{n=1}^{\infty} a_n$ converges.

(60%) (二) Choice.

(1) Find the value of the integral $\int_0^1 x\tan^{-1} dx$.

(A) $\pi/4$ (B) $\pi-2$ (C) $\pi/2$ (D) $(\pi-2)/2$

(E) $(\pi-2)/4$ (F) $\pi-1$ (G) $(\pi-1)/2$ (H) $(\pi-1)/4$

(2) Find the value of the integral $\int_0^{1/2}\sqrt{1-x^2}\,dx$.

(A) $\frac{\pi}{12}+\frac{\sqrt{3}}{8}$ (B) $\pi/4$ (C) $\pi/12+\frac{1}{4}$ (D) $\frac{\pi}{12}+\frac{\sqrt{3}}{4}$

(E) $5/24\pi$ (F) $\frac{1}{12}+\frac{\pi}{4}$ (G) $\frac{\pi}{12}-\frac{\sqrt{3}}{4}$ (H) $\frac{\pi}{12}-\frac{\sqrt{3}}{8}$

(3) Find the value of the integral $\int_0^2 \frac{1}{(x-1)^2}\,dx$.

(A) 0 (B) -2 (C) $3/2$ (D) ln 5

(E) 2 (F) $\frac{\pi}{3}$ (G) $\frac{1}{2}$ (H) diverge.

(4) Find the value of the integral $\int_0^{\pi/2} \sin^{10} x\,dx$.

(A) $\frac{1}{2}$ (B) $\frac{\pi}{3}$ (C) 0 (D) 1

(E) $\frac{63}{512}\pi$ (F) $\frac{63}{216}$ (G) 2π (H) $\frac{63}{108}\pi$

(5) Find the value of the integral $\int_0^{3/4} \frac{1}{\sqrt{x^2+1}}\,dx$.

(A) $\sqrt{3/4}$ (B) 2 (C) ln 4 (D) $\ln(5/4)$

(E) $\sqrt{5/4}$ (F) ln 2 (G) $\sqrt{2}$ (H) $\ln(3/4)$

(6) Find the shortest distance from the point (1, 4) to a point on the parabola $y^2 = 2x$.

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

(E) $\sqrt{5}$ (F) $\sqrt{6}$ (G) $\sqrt{7}$ (H) $2\sqrt{2}$

(7) Find the area of the largest rectangle that can be inscribed in the ellipse $x^2 + \frac{y^2}{4} = 1$.

(A) 1 (B) 2 (C) 3 (D) 4

(E) 10 (F) 6 (G) 7 (H) 8

(8) Find the value of the limit $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left[\left(\frac{i}{n}\right)^3 + 1\right]$.

(A) 0 (B) $1/4$ (C) $1/2$ (D) $1/3$

(E) $3/4$ (F) $2/3$ (G) 1 (H) $5/4$

(9) Let $f(x)=\int_{\sqrt{x}}^{x^2} \cos t\,dt$. Find the value of $f'(1)$.

(A) 0 (B) cos 1 (C) 2cos 1 (D) $\frac{3}{2}\cos 1$

(E) sin 1 (F) 2sin 1 (G) $\frac{3\sin 1}{2}$ (H) $\frac{3}{2}\sin 1$

(10) Find the average value of $f(x)=4\sqrt{x+1}$ on $[0,15]$.

(A) 1 (B) 56 (C) $56/3$ (D) 28

(E) $\pi/2$ (F) $56/5$ (G) 16 (H) $16/15$

(11) Which of the following series are conditionally convergent?

i) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt{n}}$ ii) $\sum_{n=1}^{\infty}\frac{n^n}{n!}$ iii) $\sum_{n=1}^{\infty}\frac{\sin n\pi}{\pi^n}$

(A) none (B) i (C) ii (D) iii
(E) i, ii (F) i, iii (G) ii, iii (H) i, ii, iii

(12) Find the value of the series $\sum_{n=1}^{\infty}\frac{1}{n2^n}$.

(A) $1/2$ (B) $\sqrt{2}$ (C) 2 (D) ln 2

(E) $1/\ln 2$ (F) $\sqrt{\ln 2}$ (G) 4 (H) 2 ln 2

(13) Find the sum of the series $\sum_{n=1}^{\infty} nx^n$.

(A) $\frac{1}{1-x}$ (B) $\frac{1}{(1-x)^2}$ (C) $\frac{x}{(1-x)^2}$ (D) $\frac{x}{1-x}$

(E) $\sqrt{1-x}$ (F) $\frac{1}{1+x}$ (G) $\frac{1}{1+x^2}$ (H) $\frac{x}{1+x^2}$

(14) Find the value of $\lim_{n\to\infty}\left(1-\frac{3}{n}\right)^{n/2}$.

(A) 1 (B) e (C) e^3 (D) e^{-3}

(E) $e^{1/2}$ (F) $e^{-1/2}$ (G) $e^{-3/2}$ (H) $e^{-2/3}$

(15) Let $f(x) = (\sin x)^x$. Find the value of $f'(\pi/2)$.

(A) $1/2$ (B) 1 (C) 0 (D) -1

(E) 2 (F) $\sqrt{2}$ (G) $-\sqrt{2}$ (H) $-1/2$

(5%) (三) Use polar coordinates to find the volume below the paraboloid $z = 4 - x^2 - y^2$ and above the xy-plane.

(15%) (四) The Gamma function, defined by $P(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$, $\alpha > 0$ has many application in both pure and applied mathematics. Show the following

(a) The integral that defines $P(\alpha)$ converges, if $\alpha > 0$

(b) $P(\alpha + 1) = \alpha P(\alpha)$, $\alpha > 0$

(c) Find the value of $P(5) = \int_0^\infty x^4 e^{-x} dx$

(d) Find the value of $P(\frac{1}{2})$

(e) Find the value of the integral $\int_0^\infty x^2 e^{-x/3} dx$.