壹. 選擇題： 下列各題均只有一個最佳答案，每題四分，共 60 分。

1. $\lim\limits_{x \to 1} x^{1/(1-x)} = ?$　　(a)∞　(b)$-\infty$　(c) e　(d) 1/e　(e) does not exist

2. $\lim\limits_{x \to 3} \dfrac{x^x - 3^3}{3^x - x^3} = ?$　　(a) 2　(b) ln2　(c) ln3　(d) 2ln2　(e) 3ln3

3. $\lim\limits_{x \to 0} \dfrac{\tan x - x}{x - \sin x} = ?$　　(a) 0　(b) 1/2　(c) 1　(d) 3/2　(e) 2

4. If $F(x) = \int_0^x e^{-x^2 t^2} dt$ then $F'(x) = ?$　　(a) $\int_0^x -2xt^2 e^{-x^2 t^2} dt$　(b) $-2xt^2 e^{-x^2 t^2}$

(c) $e^{-x^4} - \int_0^x 2xt^2 e^{-x^2 t^2} dt$　(d) $e^{-x^2 t^2} - \int_0^x 2xt^2 e^{-x^2 t^2} dt$　(e) $-2xt^2 e^{-x^2 t^2} - \int_0^x 2xt^2 e^{-x^2 t^2} dt$

5. The derivative of y^n with respect to y^3 is

(a) $\dfrac{n}{3} y^{n-3}$　(b) ny^{n-3}　(c) $3ny^{n-3}$　(d) $3ny^{n+1}$　(e) does not exist

6. The minimal point of $y = e^{-x} \sin x$ is (a) $\pi/4$　(b) $\pi/3$　(c) $\pi/2$　(d) $5\pi/4$　(e) $4\pi/3$

7. $\int_0^\infty e^{-x} \cos x\, dx = ?$　(a) 1/3　(b) 1/2　(c) $-1/2$　(d) 1　(e) -1

8. $\int_0^{\pi/2} \tan^2 \dfrac{x}{2} dx = ?$　(a) 1　(b) 2　(c) $1 - \pi/2$　(d) $2 - \pi/2$　(e) 3

9. The arc length of the curve $y = \dfrac{2}{3} x^{3/2}$ from $x = 3$ to $x = 8$ is

(a) 12　(b) 38/3　(c) 40/3　(d) 14　(e) 44/3

10. The region R enclosed by the curves $y = x$ and $y = x^2$ is rotated about x-axis, the volume of the resulting solid is　(a) $\pi/15$　(b) $2\pi/15$　(c) $\pi/5$　(d) $4\pi/15$　(e) $\pi/3$

11. The area enclosed by one loop of the four-leaved rose $r = \cos 2\theta$ is

(a) $\pi/8$　(b) $\pi/4$　(c) $\pi/3$　(d) $\pi/2$　(e) π

12. $\dfrac{d y^2}{d^2 x}$ of the function $x^{2/3} + y^{2/3} = 8^{2/3}$ is

(a) $\dfrac{4}{3x^{4/3} y^{1/3}}$　(b) $\dfrac{4}{3x^{1/3} y^{4/3}}$　(c) $\dfrac{4}{3x^{2/3} y^{2/3}}$　(d) $\dfrac{4}{3x^{1/3} y^{1/3}}$　(e) $\dfrac{4}{3x^{4/3} y^{4/3}}$

13. $f(x) = x + \dfrac{x^3}{3} + \dfrac{x^5}{5} + \dfrac{x^7}{7} + ...$, then $f(x) = ?$　(a) $(e^x + e^{-x})/2$　(b) $(e^x - e^{-x})/2$

(c) $[\ln(1+x) - \ln(1-x)]/2$　(d) $[\ln(1+x) + \ln(1-x)]/2$　(e) $(\cos x + \sin x)/2$

14 Let $f(x) = x^3 - x$, $a = 0$ and $b = 2$ the number c that satisfies the conclusion of mean value theorem is　(a) $1/\sqrt{3}$　(b) $2/\sqrt{3}$　(c) $\sqrt{3}$　(d) $1/\sqrt{2}$　(e) $\sqrt{2}$

15. (i) $\sum\limits_{1}^{\infty} \dfrac{2^n}{n!}$　(ii) $\sum\limits_{1}^{\infty} \dfrac{\sqrt{n+1} - \sqrt{n-1}}{n}$　(iii) $\sum\limits_{1}^{\infty} (-1)^n \dfrac{\ln n}{\sqrt{n}}$　which of the above series

converges ?　(a) i and ii　(b) i and iii　(c) ii and iii　(d) i, ii and iii　(e) none of them

（背面仍有題目,請繼續作答）

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貳. 計算與證明題：共 40 分

1. Let $F(y) = \int_0^\pi \sin(xy)dx$, find $F'(y)$ (10%)

2. Find the shortest distance from a given point $(0,a)$, $a > 0$, to the parabola $x^2 = 4y$ (10%)

3. Given $\Gamma(m) = \int_0^\infty x^{m-1}e^{-x}dx$ and $\Gamma(1/2) = \sqrt{\pi}$, find

    (a) $\int_0^\infty e^{-a^2x^2}dx$ (b) $E(X^{2n}) = \int_0^\infty x^{2n}e^{-a^2x^2}dx$ (c) $E(X^{2n+1}) = \int_0^\infty x^{2n+1}e^{-a^2x^2}dx$

    (d) $Var(X) = E(X^2) - (EX)^2$ (12%)

4. Prove that if $f(x)$ is a continuous function then

    (a) $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ (3%)

    (b) Use part (a) to show that $\int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x}dx = \frac{\pi}{4}$ (5%)