

一、選擇題 (一題 5 分, 共計 50 分)

1. If  $f(x) = \begin{cases} c & \text{if } x = -3 \\ \frac{9-x^2}{4-(x^2+7)^{1/2}} & \text{if } |x| < 3 \\ d & \text{if } x = 3 \end{cases}$ , then  $f$  is continuous on  $[-3, 3]$  provided
- a)  $c = 8$  and  $d = -8$    b)  $c = d = 8$    c)  $c = -8, d = 8$    d)  $c = d = 6$    e)  $c \neq d$
2. If  $f(x) = \begin{cases} |x| & \text{if } |x| \leq 1 \\ 2 - |x| & \text{if } x > 1 \end{cases}$ , then which of the following is true?
- a)  $f$  is not continuous at  $x = 1$  but it is differentiable at  $x = 1$   
 b)  $f$  is differentiable at all  $x$   
 c)  $f$  is continuous but not differentiable at  $x = 0$   
 d)  $f$  is continuous and differentiable at  $x = 1$   
 e) none of these
3. A rectangle has its vertices on the  $x$ -axis, the  $y$ -axis, the origin, and the graph of  $y = 4 - x^2$ . Find the maximum possible area for such a rectangle.
- a)  $\frac{8\sqrt{3}}{9}$    b)  $\frac{16\sqrt{3}}{9}$    c)  $\frac{32\sqrt{3}}{9}$    d)  $2\sqrt{3}$    e) none of these
4.  $\int_0^{\pi} \sin x (1 + \sqrt{\cos x})^2 dx$  equals
- a)  $\frac{17}{6}$    b) 3   c)  $\frac{19}{6}$    d)  $-\frac{19}{6}$    e) none of these
5. The area bounded by the curves  $y + x^3 = 0$ ,  $y = \sqrt{x}$ , and  $3y + 7x = 10$  equals
- a)  $\frac{53}{3}$    b)  $\frac{53}{4}$    c)  $\frac{53}{6}$    d)  $\frac{51}{5}$    e) none of these
6. The volume of the solid generated by revolving the region bounded by the curve  $y = x^3$ ,  $x = -2$ , and  $y = 0$  about the  $x$ -axis equals
- a)  $\frac{128\pi}{5}$    b)  $18\pi$    c)  $\frac{120\pi}{2}$    d)  $\frac{128\pi}{7}$    e) none of these
7. The series  $\sum_{n=1}^{\infty} \frac{1}{\ln(n)}$
- a) converges by comparison to  $a_n = \frac{1}{n}$    b) diverges by comparison to  $a_n = \frac{1}{n}$   
 c) converges by the integral test   d) converges by comparison to  $a_n = \frac{1}{n^2}$   
 e) none of these

8. The interval of convergence of  $s = \frac{x}{2^2 \cdot 3} + \frac{x^2}{3^2 \cdot 3^2} + \frac{x^3}{4^2 \cdot 3^3} + \dots$  is  
a)  $(-3, 3)$  b)  $[-3, 3)$  c)  $(-3, 3]$  d)  $[-3, 3]$  e) none of these
9. What dose  $\iint_R (x^3 - y^3) dA$  equal where  $R$  is bounded by  $y = x^3, x = 0$  and  $y = 0$ ?  
a)  $-\frac{45}{364}$  b)  $\frac{45}{364}$  c)  $\frac{9}{46}$  d)  $\frac{1}{5}$  e) none of these
10. The volume of the solid in the first octant bounded by  $x + y = 4$  and  $z = xy$  is  
a)  $\frac{44}{3}$  b) 15 c)  $\frac{42}{3}$  d)  $\frac{43}{3}$  e) none of these

二、計算或證明題 (一題 10 分, 共計 50 分), 需寫出詳細計算或證明過程, 否則扣分或不計分。

- Using the Definition of a Limit, Show that  $\lim_{x \rightarrow c} \frac{1}{x} = \frac{1}{c}, c \neq 0$ .
- Show that  $f(x) = \sqrt{2x+1} + 2x$  is continuous at 3.
- Show that  $f$  satisfies the hypotheses of Rolle's theorem on the interval  $[2, 6]$ , and find all numbers  $c$  in  $(2, 6)$  that satisfy the conclusion of Rolle's theorem for  $f(x) = 2x^2 - 16x + 11$ .
- A solid has as its base the region in the  $xy$ -plane bounded by the graphs of  $y^2 = 4x$  and  $x = 4$ . If every cross section by a plane perpendicular to the  $y$ -axis is a semicircle, find the volume of the solid.
- Express  $\iint_R (3x + 2y) dA$  over the triangular region with vertices  $(-2, -2), (-1, -2), (-1, -1)$  as an iterated integral and find its value.