

Please write down all your work.

1. Evaluate the given limits, if exists. (10%)

$$(a) \lim_{x \rightarrow \frac{\pi}{2}^+} \cos x \log(\cot x)$$

$$(b) \lim_{x \rightarrow 0^+} \left(1 + \frac{x}{3} + \frac{x^2}{5}\right)^{\frac{2}{x}}$$

2. Find the following integrals, if exist. (30%)

$$(a) \int_1^3 \frac{x}{(x^2 - 4)^3} dx$$

$$(b) \int_0^{\infty} y^3 e^{-\sqrt{y}} dy$$

$$(c) \int_0^1 x^7 (1-x)^{100} dx$$

$$(d) \int_0^{\frac{\pi}{4}} \tan^5 4x \sec^2 4x dx$$

$$(e) \int_0^{\frac{1}{2}} (\cos^{-1} x) dx$$

$$(f) \int_{-1}^1 \frac{1}{\sqrt{x^2 - 2x + 26}} dx$$

3. Determine whether the given series converges absolutely, converges conditionally, or diverges. (10%)

$$(a) \sum_{n=1}^{\infty} \frac{\cos n}{e^n}$$

$$(b) \sum_{n=1}^{\infty} \frac{n!}{(-\sqrt{n})^n}$$

4. Find the Maclaurin Series for $f(x) = xe^x$ and find the sum of $\sum_{n=0}^{\infty} \frac{n+1}{n!}$. (10%)

5. Convert

$$\int_0^5 \int_0^{\sqrt{25-y^2}} e^{\sqrt{x^2+y^2}} dx dy$$

to a double integral in polar coordinates and evaluate the integral. (10%)

6. Let Q be the region in the xy -plane bounded above by the parabola $x = y^2$ and below by the line $x = 3y$. Evaluate $\iint_Q xy dA$. (10%)

7. Let

$$F(x) = \int_{-x}^x \frac{1}{|t|+1} dt, \quad |x| < 1$$

Find the derivative of $F(x)$, if exists. (10%)

8. Find the interval of convergence of the given power series. (10%)

$$(a) \sum_{k=1}^{\infty} \frac{\ln k}{e^k} (x - e)^k$$

$$(b) \sum_{k=1}^{\infty} \frac{(k!)^2}{(2k)!} x^k$$