93學年度國立成功大學研究所招生考試 國際企業研究所 微積分 試題 共 1 頁 第 1 頁

Please write down all your work.

1. Show that $\int_0^1 \cos\frac{1}{x}dx$ exists. (10%)

2. Find the derivative, if exists. (20%)

(a) $y = e^{e^x}$

(b) $y = \cos x^{\tan x}$

(c) $y = \tan^{-1}(\sinh x)$

(d) $f(x) = \int_{\ln x}^{x^2} \ln t \; dt$

3. Evaluate the given integral. (20%)

(a) $\int \sec^3 3x dx$

(b) $\int \frac{2x^3 + x^2 + 2x - 1}{x^4 - 1} dx$

(c) $\int \frac{1}{\sqrt{29 - 4x + x^2}} dx$

(d) $\int_1^5 \frac{x}{\sqrt{(x^2 - 9)}} dx$

4. Determine whether the given series converges or diverges. (10%)

(a) $\sum_{n=0}^{\infty} n^2 e^{-n} \sin n$

(b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

5. Find the sum of the given series. (10%)

(a) $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n}$

(b) $\sum_{n=0}^{\infty} \frac{n^2}{3^n}$

6. Find the length of the curve $2y = 3(x+1)^{2/3}$ from (-2, 3/2) to (7, 6). (10%)

7. If n is a positive integer, show that $y = x^n e^{-x}$ assumes its maximum value at $x = n$, so that its value at $x = n - 1$ and $x = n + 1$ are less than the maximum. Use this fact to show that $(\frac{n+1}{n})^n < e < (\frac{n}{n-1})^n$; and use this to show that $(1 + \frac{1}{n})^n < e < (1 + \frac{1}{n})^{n+1}$ for every n. Moreover, show that $\lim_{n\to\infty}(1 + \frac{1}{n})^n = e$ and $2 < e < 3$. (20%)

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