

本試題是否可以使用計算機: 可使用, 不可使用 (請命題老師勾選)

SHOW ALL YOUR WORK!

1. Evaluate the following integrals, if exist. (20%)

(a) $\int_0^5 |x^3 - x| dx$

(b) $\int_0^{\pi/2} \int_x^{\pi/2} \frac{\sin y}{y} dy dx$

(c) $\int_0^1 x^5 (1-x^3)^{1/3} dx$

(d) $\int_1^{\infty} \frac{1}{e^x - 1} dx$

2. Let $F(x) = \int_0^1 \sin(xy^2) dy$. Find the Taylor polynomial for F of degree 3. (10%)

3. If D is the region in the xy -plane bounded by the curves $y = x + 6$ and $y = x^2$, then evaluate $\iint_D x dx dy$. (10%)

4. Find the following limits, if exist. (20%)

(a) $\lim_{x \rightarrow \infty} \left[\ln x - \ln(x + \sqrt{x^2 + 1}) \right]$

(b) $\lim_{n \rightarrow \infty} \frac{\ln(3n)}{3^n}$

(c) $\lim_{n \rightarrow \infty} \left(n \cdot \sin \frac{\pi}{2n} \right)$

(d) $\lim_{x \rightarrow 2} \frac{3^{x/2} - 3}{3^x - 9}$

5. What is the maximum value of $f(x, y) = 2x + y^2$ on the circle $x^2 + y^2 = 9$, and occurred at which point? (10%)

6. If $f(x) = \sum_{n=0}^{\infty} \frac{x^{n+1}}{2(n+1)}$ for all x in $(-1, 1)$, then $f'(1/10) = ?$ (10%)

7. If $\sum_{n=2}^{\infty} \left(\frac{1}{1+a} \right)^n = 3$ and $a > 0$, then $a = ?$ (10%)

8. If $\lim_{t \rightarrow 0} \frac{f(\sin(t) + 2t) - f(0)}{t} = 0.25$, find the value of $f'(0)$? (10%)