編號： 398　系所：國際企業研究所　　科目：微積分

本試題是否可以使用計算機： □可使用 ， ☑不可使用 （請命題老師勾選）

Please show all your work.

1. (10%) Find $\lim_{x\to 0}\frac{1}{x}\ln\left(\frac{a^x+b^x+c^x+d^x}{4}\right)$, if a>0, b>0, c>0, d>0.

2. (10%) Kenny is standing on Taipei 101, which is a 101-floor landmark skyscraper located in Taipei. Through a telescope, he is watching a car approaching the building directly below him. Assume the telescope is 500 meters above the ground level and the car is approaching at 1,000 meters per minute. At what rate is the angle of the telescope changing when the car is 500 meters away from Taipei 101?

3. (10%) (a) Evaluate $\int_0^{\frac{\pi}{4}}\frac{1}{1+\sin x}dx$; (b) Find $\lim_{n\to\infty}\frac{\frac{1}{\sqrt{1}}+\frac{1}{\sqrt{2}}+\cdots+\frac{1}{\sqrt{n}}}{\sqrt{n}}$

4. (10%) Find the area of the region between the line $3y-4x+4=0$ and the parabola $y^2=4x$.

5. (10%) Let $f(x)=\int_0^x\frac{e^{t^2}-1}{t^2}dt$, find $f^{(3)}(0)$.

6. (10%) (a) Evaluate $\int_0^2\int_{\frac{y}{2}}^1 e^{x^2}dxdy$;

   (b) Evaluate $\iint_A xydxdy$; $A: y\geq 4, x^2+y^2\leq 25$

7. (10%) Find the minimum value of $f(x,y,z)=3x+2y+z+\frac{1}{2}$ subject to the constraint $g(x,y,z)=9x^2+4y^2-z=0$.

8. (10%) Find the general solution to $y''-5y'+12.5y=0$.

9. (10%) Find the volume of the solid generated by revolving the region bounded by the curve $y=\sqrt{x}$, the x-axis, and the line $x=5$ about the x-axis.

10. (10%) Determine the convergence or divergence of the following series: $\sum_{n=1}^{\infty}\frac{n!}{n^n}$.