

系所組別： 國際企業研究所乙組

考試科目： 微積分

考試日期： 0308，節次： 3

※ 考生請注意：本試題 可 不可 使用計算機

Please show all your work.

1. (10%) A firm finds that its cost and demand functions can be modeled by the following polynomials, $C(x) = -\frac{x^2}{1000} + 5x + 3800$ and $p(x) = -\frac{x}{100} + 50$, respectively, where x refers to the production level. What production level will maximize profits for this firm?
2. (10%) Find the area of the region bounded by the curves $y = \sin x$ and $y = \cos x$, $x=0$, and $x = \frac{\pi}{2}$.
3. (10%) The curve $y = \sqrt{4-x^2}$, $-1 \leq x \leq 1$, is an arc of the circle $x^2 + y^2 = 4$. Find the area of the surface obtained by rotating this arc about the x -axis. (The surface is a portion of a sphere of radius 2.)
4. (10%) Evaluate $\int \frac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$
5. (10%) Evaluate $\int_0^4 \frac{1}{x-1} dx$
6. (10%) Find $\int_a^a \int_{\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2} dy dx$
7. (10%) Find the general solution to $y'' - y = e^x(x^2 - 1)$
8. (10%) Find the maximum value of $6x + 3y + 2z - 5$ on the surface $4x^2 + 2y^2 + z^2 = 70$
9. (10%) Find $\lim_{\lambda \rightarrow 0} \int_{\lambda}^{2\lambda} \frac{e^{-x}}{x} dx$
10. (10%) Find $\lim_{x \rightarrow 0^+} \frac{\sqrt{1+\sin x} - \sqrt{1-\sin x}}{x}$