系所組別： 體育健康與休閒研究所乙組
考試科目： 微積分
考試日期：0220，節次：3

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(50%) Part A. 單一選擇題(一題 5 分，答錯不倒扣。不熟悉微積分者應從計算題答起)

1. Suppose $L=\lim_{x\to 3}\frac{x+4}{x^2-1}$. This means that we want to calculate $\frac{x+4}{x^2-1}$ when x approaches 3. Thus, L is equal to (a) 1/2 (b) 3/4 (c) 7/8 (d) 8/9

2. Suppose $L=\lim_{x\to -5}\frac{x^2+x-20}{x+5}$. In this case, L is (a) 0 (b) −1 (c) −9 (d) undefined (it cannot be calculated).

3. Suppose $L=\lim_{x\to 0^+}\frac{e^x-1}{x}$. In this case, L is (a) 0 (b) 1 (c) −1 (d) undefined.

4. Suppose f(x) $=\frac{1}{x}$. Then $\frac{df(x)}{dx}$ is (a) $\frac{1}{x}$ (b) x (c) −x (d) $\frac{-1}{x^2}$

5. For the expression $\sqrt{x}+\sqrt[3]{y}=5$, the derivative $\frac{dy}{dx}$ can be found implicitly as (a) $-\frac{3}{2}\frac{y^{2/3}}{x^{1/2}}$ (b) $\frac{3}{2}\frac{y^{2/3}}{x^{1/2}}$ (c) $-\frac{1}{2}\frac{y^2}{x^{1/2}}$ (d) $\frac{y}{x}$

6. Suppose f(x) $=x^x$. Then $\frac{df(x)}{dx}$ is (a) $\frac{1}{x}+x$ (b) $x^x(1+\ln x)$ (c) $-x\ln x$ (d) x^x

7. Suppose f(x) $=x+3x^2$. Then $\int f(x)dx$ is (a) x (b) x(1+x) (c) x^2+x^3+C (d) $\frac{1}{2}x^2+x^3+C$. Here C is a constant.

8. For x>0, the area enclosed by $f(x)=x^3$ and $f(x)=x$ is (a) 1 (b) 1/4 (c) 3/4 (d) 1/2.

9. Suppose f(x) $=\frac{1}{x^2\sqrt{16-x^2}}$. Then $\int f(x)dx$ is (a) $\frac{\cot x}{16}+C$ (b) $\frac{-\cot x}{16}+C$ (c) $\frac{-\cos x}{16}+C$ (d) $\frac{-\tan x}{16}+C$. Here C is a constant.

10. Suppose f(x) $=xe^x$. Then $\int f(x)dx$ is (a) x+C (b) xe^x+C (c) xe^x+e^x+C (d) xe^x-e^x+C. Here C is a constant.

(背面仍有題目,請繼續作答)

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(50%) Part B. 計算題 (共 3 題)

1. The definition of $\frac{df(x)}{dx}$, which is called derivative of f(x) with respect to x, is the rate of change of f(x) for an infinitely small change in x. That is, $\frac{df(x)}{dx}$ is defined as

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{x+h-x}=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

(a) Please use the above definition to calculate $\frac{df(x)}{dx}$, where $f(x) = x^2$ (此題 5 分)

(b) Use the definition to show that the derivation of f(x)=|x| does not exist because $\frac{df(x)}{dx}$ calculated for positive h is different from $\frac{df(x)}{dx}$ calculated for negative h. (此題 5 分)

2. Suppose we have a differential equation $\frac{dy}{dt}=2$. To solve for y we can bring dt to the right side to have dy = 2 dt. Then integrate both sides to get y = 2t + C, where C is a constant.

(a) Please use the above definition to solve for y in $\frac{dy}{dt}=\sin t + t$ (此題 5 分)

(b) A free-fall body's motion is governed by the differential equation $\frac{d^2y}{dt^2}=g$, where g is the gravitational acceleration. Please solve for y (此題 10 分)

(c) If we know the initial condition of y, then C can be evaluated. For example, if we know that y=1 at t=0, then from y = 2t + C we know that 1 = 0 + C, which means C =1. Suppose we know that y=Y at t=0 and $\frac{dy}{dt}=V$ at t=0. Use these initial conditions to show that the equation $\frac{d^2y}{dt^2}=g$ can be solved to get $y = Y + Vt + gt^2/2$. (此題 5 分)

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3. The velocity of a point in reference frame N is defined as taking the derivative of its position vector with respect to time in frame N. For example if the position vector $\vec{P} = 2t\hat{n}_1 + \sin t\,\hat{n}_2$, then the velocity is $2\hat{n}_1 + \cos t\,\hat{n}_2$. Here $\hat{n}_1$ and $\hat{n}_2$ are unit vectors fixed in frame N, and therefore do not change with time.

(a) A disk (called body B) is rotating relative to the fixed reference frame N. Pont O is fixed in N and also in B. Another point Q has the position vector $R\hat{b}_1 = R(\cos\omega t\,\hat{n}_1 + \sin\omega t\,\hat{n}_2)$ relative to point O. Here R and ω (angular speed) are constants. This means that Q is fixed in frame B but is moving in frame N. Calculate the velocity of Q in frame B and in frame N. (此題 10 分)

(b) From the figure we know $\hat{b}_1 = \cos\omega t\,\hat{n}_1 + \sin\omega t\,\hat{n}_2$, and $\hat{b}_2 = -\sin\omega t\,\hat{n}_1 + \cos\omega t\,\hat{n}_2$.

Show that the cross product $\omega\hat{b}_3 \times R\hat{b}_1$ is equal to the velocity of Q in frame N. (此題 5 分)

(c) Actually if body B is rotating in N with the angular velocity $\bar{\omega} = \omega\hat{b}_3 = \omega\hat{n}_3$, then taking time derivative of a vector $\vec{P}$ in frame N is equal to taking time derivative in frame B plus the term $\bar{\omega} \times \vec{P}$. With this knowledge, if $\vec{P} = t^2\hat{b}_1$, calculate the velocity in frame N. (此題 5 分)