國立成功大學

114學年度碩士班招生考試試題

編　號：188

系　所：數據科學研究所

科　目：統計學

日　期：0210

節　次：第 1 節

注　意：1.不可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (15%) 成大醫院為提升醫療服務的品質，並瞭解病人對本院醫療服務品質的滿意程度，分別針對門診服務、住院服務和急診服務三個項目進行滿意度調查。依接受醫療服務民眾的滿意程度整理如下表

滿意程度	門診服務	住院服務	急診服務
滿意	800	600	600
不滿意	200	200	100

(1) 醫療服務和民眾的滿意程度是否獨立？請用機率獨立事件方式驗證。(5%)

(2) 醫療服務三個項目的民眾滿意度是否相同，請用假設檢定方式驗證，型 I 誤差設為 0.05。(5%)

(3) 從表中可以估計出在住院服務的民眾中大約有 200/(200+600)=0.25 的機率感到不滿意。今有一位對醫療服務不滿意的民眾到院投訴，試問這位民眾是對住院服務感到不滿意的機率是多少？請使用貝氏定理計算。(5%)

2. (15%) 手機大廠的品質管制問題。在製程的某個階段必須進行破壞性檢測(但是可以修復)，所以抽檢的數量必須很少。以往該階段的良率是 0.99。假設該批次的數量是 10 萬支手機。

(1) 如果測試方式是隨機抽三支手機，隨機變數 X 定義為缺陷手機數量，其中只有一支手機有缺陷(X=1)的機率為何? (5%)

(2) 如測試方式是持續抽取手機進行測試，直到一支手機有缺陷才停止，隨機變數 Y 定義為抽取檢測的手機數量。檢測到第三支手機有缺陷(Y=3)的機率為何? (5%)

(3) 隨機抽三支手機，隨機變數 X 定義為缺陷手機數量，檢測結果 X=1，在型 I 誤差為 0.05 之下，檢定虛無假設良率 0.99 是否該被拒絕，對立假設為良率低於 0.99。(5%)

3. (20%) 現今是大數據的時代，四處都有巨量的資料需要進行適當的分析。假設以下觀察到的大量資料具有隨機變數的性質，可以用 n 個隨機變數記為$X_1, X_2, \ldots, X_n$. 且這些隨變數都具有相同期望值 μ。

(1) 請完整敘述中央極限定理。(5%)

(2) 請敘述中央極限定理在統計推論上假設檢定應用於檢定μ = 0方法，在型 I 誤差為 0.05 之下，請以計算公式表示假設檢定做法。(5%)

(3) 2024 年每個月大約有 2 億 ChatGP 使用者，Instagram 大約 20 億活躍使用者。今在 Instagram 發問卷，詢問喜歡或不喜歡 ChatGP，取得 100 萬人的回答。想檢定喜歡 ChatGP 的比率是否大於 0.5。可以採取中央極限定理在統計推論上假設檢定的應用方法嗎? 請解釋可以或不可以的原因。(10%)

4. (10%)國家競爭力分為 4 大指標，分別是「企業效能」、「政府效能」、「基礎建設」、「經濟表現」。

下表為 10 個亞洲國家 2010 年及 2020 年的國家競爭力排名：

國　家	2010 年排名	2020 年排名
台　灣	22	18
新加坡	2	2
香　港	14	7
日　本	17	16
馬來西亞	25	27
韓　國	28	38
大　陸	31	29
泰　國	33	34
菲律賓	38	32
印　尼	45	46

(1) 試求 2010 年及 2020 年的國家競爭力排名的 Spearman 等級相關係數。(5%)

(2) 在$\alpha = 0.01$下，以等級相關檢定法檢定 2010 年及 2020 年的國家競爭力排名是否為正相關？(5%)

5 (25%)考慮以下簡單線性回歸模型來描述自變數 x 和應變數 y 的關係

$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i, \mathrm{i} = 1,2,..\mathrm{n}$$

(1) 請用最小平方法求 β_0, β_1的估計量$\hat{\beta}_0, \hat{\beta}_1$，請列出計算過程。(5%)

(2) 在$\varepsilon_i \sim iid\ N(0,\sigma^2)$常態假設之下，$\frac{\mathrm{SSE}}{\sigma^2} \sim \chi^2(n-2)$分布。使用這些性質求算 E(MSE)。(5%)

(3) 計算 $\mathrm{cov}(\bar{y}, \hat{\beta}_1)$。(5%)

(4) 寫出 ANOVA 表，說明如何在型 I 誤差為 0.05 之下檢定$\beta_1 = 0$。(5%)

(5) 如何檢查ε_i是否為常態分佈？請詳細列出兩種方法，包含計算公式。(5%)

6 (15%)考慮以下以矩陣表式的多元回歸模型來描述自變數x_{1i}, x_{2i}和應變數y_i的關係

$$\boldsymbol{y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon}$$

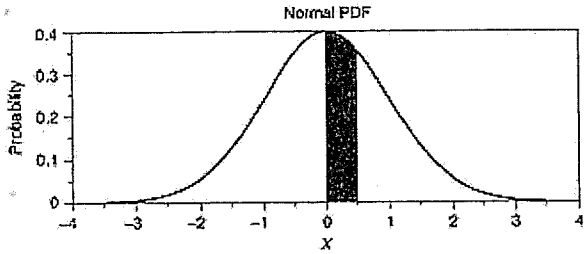
這裡 $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)^t$, $\boldsymbol{\beta} = (\beta_0, \beta_1, \beta_2)^t$, $\boldsymbol{X} = \begin{pmatrix} 1 & x_{11} & x_{21} \\ 1 & x_{12} & x_{22} \\ \vdots & \vdots & \vdots \\ 1 & x_{1n} & x_{2n} \end{pmatrix}$, $\boldsymbol{\varepsilon} = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)^t \sim N(0, \sigma^2 \boldsymbol{I})$, $\boldsymbol{I}$ 是 nxn 的對角單位矩陣。

(1) 請用最小平方法和矩陣表示式，求$\boldsymbol{\beta}$的估計量$\hat{\boldsymbol{\beta}}$，請列出計算過程。(5%)

(2) 求取估計量$\hat{\boldsymbol{\beta}}$的期望值。(5%)

(3) 當 $\boldsymbol{X}$矩陣的行向量是線性相依，需如何處理以建立適當回歸模型。(5%)

Normal Table



Area under the Normal Curve from 0 to X

X	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998

Chi-Square Table

df	$\alpha = 0.995$	$\alpha = 0.99$	$\alpha = 0.975$	$\alpha = 0.95$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$
1	0.000	0.000	0.001	0.004	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	11.070	12.832	15.086	16.750
6	0.676	0.872	1.237	1.635	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	14.067	16.013	18.475	20.278
8	1.344	1.647	2.180	2.733	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	36.415	39.364	42.980	45.558
25	10.520	11.524	13.120	14.611	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	38.885	41.923	45.642	48.290
27	11.808	12.878	14.573	16.151	40.113	43.195	46.963	49.645
28	12.461	13.565	15.308	16.928	41.337	44.461	48.278	50.994
29	13.121	14.256	16.047	17.708	42.557	45.722	49.588	52.335
30	13.787	14.953	16.791	18.493	43.773	46.979	50.892	53.672
31	14.458	15.655	17.539	19.281	44.985	48.232	52.191	55.002
32	15.134	16.362	18.291	20.072	46.194	49.480	53.486	56.328
33	15.815	17.073	19.047	20.867	47.400	50.725	54.775	57.648
34	16.501	17.789	19.806	21.664	48.602	51.966	56.061	58.964
35	17.192	18.509	20.569	22.465	49.802	53.203	57.342	60.275
36	17.887	19.233	21.336	23.269	50.998	54.437	58.619	61.581
37	18.586	19.960	22.106	24.075	52.192	55.668	59.893	62.883
38	19.289	20.691	22.878	24.884	53.384	56.895	61.162	64.181
39	19.996	21.426	23.654	25.695	54.572	58.120	62.428	65.475
40	20.707	22.164	24.433	26.509	55.758	59.342	63.691	66.766
50	27.991	29.707	32.357	34.764	67.505	71.420	76.154	79.490
60	35.534	37.485	40.482	43.188	79.082	83.298	88.379	91.952
70	43.275	45.442	48.758	51.739	90.531	95.023	100.425	104.215
80	51.172	53.540	57.153	60.391	101.879	106.629	112.329	116.321
90	59.196	61.754	65.647	69.126	113.145	118.136	124.116	128.299
100	67.328	70.065	74.222	77.929	124.342	129.561	135.807	140.170

Spearman Table

	Level of Significance of a One-Tailed or Directional Test H_0: $\rho_S \leq 0$ or H_0: $\rho_S \geq 0$					
	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.025$	$\alpha = 0.01$	$\alpha = 0.005$	$\alpha = 0.0005$
	Level of Significance of a Two-Tailed or Nondirectional Test H_0: $\rho_S = 0$					
df	$\alpha = 0.2$	$\alpha = 0.1$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	$\alpha = 0.001$
2	1.000	1.000	—	—	—	—
3	0.800	0.900	1.000	1.000	—	—
4	0.657	0.829	0.886	0.943	1.000	—
5	0.571	0.714	0.786	0.893	0.929	1.000
6	0.524	0.643	0.738	0.833	0.881	0.976
7	0.483	0.600	0.700	0.783	0.833	0.933
8	0.455	0.564	0.648	0.745	0.794	0.903
9	0.427	0.536	0.618	0.709	0.755	0.873
10	0.406	0.503	0.587	0.678	0.727	0.846
11	0.385	0.484	0.560	0.648	0.703	0.824
12	0.367	0.464	0.538	0.626	0.679	0.802
13	0.354	0.446	0.521	0.604	0.654	0.779
14	0.341	0.429	0.503	0.582	0.635	0.762
15	0.328	0.414	0.485	0.566	0.615	0.748
16	0.317	0.401	0.472	0.550	0.600	0.728
17	0.309	0.391	0.460	0.535	0.584	0.712
18	0.299	0.380	0.447	0.520	0.570	0.696
19	0.292	0.370	0.435	0.508	0.556	0.681
20	0.284	0.361	0.425	0.496	0.544	0.667
21	0.278	0.353	0.415	0.486	0.532	0.654
22	0.271	0.344	0.406	0.476	0.521	0.642
23	0.265	0.337	0.398	0.466	0.511	0.630
24	0.259	0.331	0.390	0.457	0.501	0.619
25	0.255	0.324	0.382	0.448	0.491	0.608
26	0.250	0.317	0.375	0.440	0.483	0.598
27	0.245	0.312	0.368	0.433	0.475	0.589
28	0.240	0.306	0.362	0.425	0.467	0.580
29	0.236	0.301	0.356	0.418	0.459	0.571
30	0.232	0.296	0.350	0.412	0.452	0.563

Student's T Table

	Level of Significance of a One-Tailed or Directional Test H_0: $\mu - \mu \geq 0$ or H_0: $\mu - \mu \leq 0$					
	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = .025$	$\alpha = 0.01$	$\alpha = 0.005$	$\alpha = 0.0005$
	$1-\alpha = 0.90$	$1-\alpha = 0.95$	$1-\alpha = 0.975$	$1-\alpha = 0.99$	$1-\alpha = 0.995$	$1-\alpha = 0.9995$
	Level of Significance of a Two-Tailed or Nondirectional Test H_0: $\mu - \mu = 0$					
	$\alpha = 0.20$	$\alpha = 0.10$	$\alpha = 0.05$	$\alpha = 0.02$	$\alpha = 0.01$	$\alpha = 0.001$
df	$1-\alpha = 0.80$	$1-\alpha = 0.90$	$1-\alpha = 0.95$	$1-\alpha = 0.98$	$1-\alpha = 0.99$	$1-\alpha = 0.999$
1	3.078	6.314	12.706	31.821	63.656	636.578
2	1.886	2.920	4.303	6.965	9.925	31.600
3	1.638	2.353	3.182	4.541	5.841	12.924
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.160	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.689
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.660
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
50	1.299	1.676	2.009	2.403	2.678	3.496
60	1.296	1.671	2.000	2.390	2.660	3.460
70	1.294	1.667	1.994	2.381	2.648	3.435
80	1.292	1.664	1.990	2.374	2.639	3.416
90	1.291	1.662	1.987	2.368	2.632	3.402
100	1.290	1.660	1.984	2.364	2.626	3.390
150	1.287	1.655	1.976	2.351	2.609	3.357
200	1.286	1.653	1.972	2.345	2.601	3.340
500	1.283	1.648	1.965	2.334	2.586	3.310
1000	1.282	1.646	1.962	2.330	2.581	3.300
∞	1.282	1.645	1.960	2.326	2.576	3.290