系所組別： 土木工程學系甲、乙、丁組

考試科目： 工程數學

考試日期：0219，節次：3

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1. (10%) The steady-state temperature in a region bounded by two long coaxial cylinders is

$$\frac{d}{dr}\left(r\frac{dU}{dr}\right) = 0,$$

where U is the temperature at distance r from the common axis. Suppose that the temperature U is kept at constant temperature T_1 and T_2, respectively, on $r = a$ and $r = b$, where $a < b$. Derive the temperature field $U(r)$.

2. (10%) Solve

$$y' = \frac{xy^2 - 1}{1 - x^2 y}, \text{ with } y(0) = 1$$

3. (10%) Evaluate the surface integral

$$\iint_S \boldsymbol{F} \cdot \boldsymbol{n} dS,$$

using the divergence theorem, where $\boldsymbol{F}(x, y, z) = \sin y\boldsymbol{i} + e^x\boldsymbol{j} + z^2\boldsymbol{k}$ and S consists of the surfaces described by $z = \sqrt{a^2 - x^2 - y^2}$ and $z = 0$.

4. (10%) Evaluate the line integral

$$\int_C \boldsymbol{F} \cdot d\boldsymbol{x},$$

where $\boldsymbol{F}(x, y, x) = (x + y^2)\,\boldsymbol{i} + (x + z)\,\boldsymbol{j} + xy\boldsymbol{k}$ and C is the path from the origin to the point $(1, -1, 1)$ along the curve

$$C : \boldsymbol{x} = t\boldsymbol{i} - t^2\boldsymbol{j} + t^3\boldsymbol{k},\ (0 \le t \le 1).$$

(背面仍有題目,請繼續作答)

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5. (10%) Given

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 3 & 1 & 1 \\ 2 & 3 & 1 \end{bmatrix}$$

If we have $A^{-1} = \alpha_0 I + \alpha_1 A + \alpha_2 A^2$, find the coefficients α_0,α_1,α_2.

6. (10%) The following set of equations possesses a one-parameter family of solutions:

$$\begin{aligned} 2x_1 - x_2 - x_3 &= 2, \\ x_1 + 2x_2 + x_3 &= 2, \\ 4x_1 - 7x_2 - 5x_3 &= 2. \end{aligned}$$

Find the general solution.

7. (10%) Evaluate

$$\oint_C \frac{3z^3 + 2}{(z-1)(z^2+9)} dz,$$

C is taken counterclockwise around the circle: $|z - 2| = 2$.

8. (10%) Use residues theorem to calculate the integral

$$\int_{-\pi}^{\pi} \frac{d\theta}{1 + \sin^2\theta}$$

9. (20%) Apply separation of variables to solve the partial differential equation

$$\begin{aligned} \nabla^2 u(x, y) &= 0, \text{ for } 0 < x < a,\ 0 < y < b, \\ u(x, 0) &= 0 \text{ for } 0 \le x \le a, \\ u(0, y) &= u(a, y) = 0 \text{ for } 0 \le y \le b, \\ u(x, b) &= f(x) \text{ for } 0 \le x \le a. \end{aligned}$$