1. The cumulative distribution function of a random variable X is given as follows. (1) Determine the probability mass function of X, and (2) compute $P(2 < X \le 4)$. (3 points each)

$$F(x) = \begin{cases} 0, & x < -2 \\ 1/2, & -2 \le x < 2 \\ 2/3, & 2 \le x < 4 \\ 5/6, & 4 \le x < 6 \\ 1, & x \ge 6. \end{cases}$$

2. The reaction time (in seconds) to a certain stimulus is a continuous random variable with pdf as follows: (3 points each)

$$f(x) = \begin{cases} 3/(2x^2) & 1 \le x \le 3 \\ 0, & \textit{otherwise} \end{cases}$$

(1) Obtain the cdf.

(2) What is the probability that reaction is at most 2.5 sec? Between 1.5 and 2.5 sec?

(3) Compute the expected reaction time. [Note that ln(3) =1]

(4) Compute the variance of reaction time.

3. Consider the function $f(x) = C \cdot (2x^2 - x^3)$ if $0 \le x \le 3$ and 0 otherwise. Could this function be a probability density function? (1) If so, determine C. (2) Repeat if this function were given by $f(x) = C \cdot (2x^2 - x^3)$ if $0 \le x \le 1$ and 0 otherwise. (6 points each)

4. Suppose that $2\hat{\theta}_1$, $\hat{\theta}_2/5$ and $\hat{\theta}_3/4$ are the unbiased estimators of the parameter θ. We know that $E(2\hat{\theta}_1) = E(\hat{\theta}_2/5) = E(\hat{\theta}_3/4) = \theta$, $Var(\hat{\theta}_1) = 10$, $Var(\hat{\theta}_2) = 12$ and $Var(\hat{\theta}_3) = 16$. Which of the following is the minimum variance unbiased estimator for θ? (1) $\hat{\theta}_2/5$ (2) $\hat{\theta}_3/4$ (3) $\hat{\theta}_1/2$ (4) $2\hat{\theta}_1$ (5) $\hat{\theta}_2$. Show your calculation to get the credits. (4 points)

5. In a study published in the Car and Driver magazine on the top of five supercars, a portion of the following ANOVA table was obtained. (1) Complete the ANOVA table, (2) State the null and alternative hypotheses needed for the analysis to be valid. (3) Test the hypothesis and state your conclusion. Use α=0.05. (5 points each)

Source	DF	SS	MS	F
Car	?	2000	?	?
Error	25	?	?	
Total	?	2500		

Values Provided for Your Calculations

z	-1.00	1.64	1.96	2	2.5	3	$t_{0.05,49}$	$t_{0.1,49}$	$t_{0.05,50}$	$F_{0.05,4,25}$	$F_{0.05,5,26}$	$F_{0.05,4,26}$
Φ(z)	0.159	0.95	0.975	0.977	0.994	0.998	1.676	1.299	2.009	2.76	2.57	2.74

6. In order to determine whether good looks translate into heftier paychecks, an engineer collected data on the annual income of 25 doctors (y) in hundreds of thousands of dollars and attractiveness (x) as recorded on a scale from 1 to 5, based on a panel's rating of head-and-shoulder photographs. The following summary values were calculated:: $\Sigma x = 75$, $\Sigma x^2 = 275$, $\Sigma y = 60$, $\Sigma y^2 = 164$, $\Sigma xy = 200$
   (1) Calculate S_{xx}, S_{yy}, and S_{xy}. (3 points)
   (2) Fit a linear regression $y = \beta_0 + \beta_1 x$ using the above data (2 points)
   (3) Compute the ANOVA table for the regression. (8 points)
   (4) Calculate the coefficient of determination and give an interpretation in the context of the problem. (5 points)

7. Answer the following questions. (3 points each)
(1) An experiment on the teaching of reading compares two methods, A and B. The response variable is the Degree of Reading Power (DRP) score. The experimenter uses Method A in a class of 26 students and Method B in a comparable class of 24 students. The classes are assigned to the teaching methods at random. Suppose that in the population of all children of this age the DRP score has the $N(34,12^2)$ distribution if Method A is used and the $N(37,11^2)$ distribution if Method B is used. Let mean DRP score for 26 students in the A group be $\overline{X}$, and mean DRP score for 24 students in the B group be $\overline{Y}$. What is the distribution of $\overline{Y} - \overline{X}$? (a) $N(3,11^2 - 12^2)$, (b) $N(3,(11+12)^2)$, (c) $N(3,(11/\sqrt{24} + 12/\sqrt{26})^2)$, (d) $N(3,(11^2/24 - 12^2/26))$, (e) $N(3,(11^2/24 + 12^2/26))$

Use the information in the following setting to answer questions 2 through 4.

The weights of samples of six tubers of four varieties of potatoes (BUR, KEN, NOR, RLS) grown under specific laboratory conditions are compared as follows.

Analysis of Variance for weight

Source	DF	SS	MS	F	P
Variety	-	-	-	4	0.25
Error	-	-	25		
Total	-	-			

(2) Which of the following hypothesis should be used for testing the differences between those four varieties? (a) H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4$ (b) H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4 = 0$ (c) H_0: $\mu_1 = \mu_2 = \mu_3 = \mu_4 = 0$ (d) H_0: $\beta_1 = \beta_2 = \beta_3 = \beta_4$ where β_i is the coefficient of that variable, i=1,2,3,4

(3) Which of the following is the test statistics for testing the average differences between those four varieties? (a) 0.25 (b) 5 (c) 0.5 (d) 4 (e) 25

(4) Which of the following is the point estimate for the constant standard deviation, σ? (a) 0.25 (b) 5 (c) 0.5 (d) 4 (e) 25.

(5) The Pedestrian Safety Committee wants to prove that the number of accidents involving pedestrians has decreased since a new crosswalk was added to Main Street. The Committee decides it wants to set $\alpha = 0.05$. What does that mean in this case? (a) The probability that, over the long run, the Committee finds a test statistic of 0.05 or greater is α. (b) 0.05 is the probability that the Committee concludes safety has been improved when in reality Main Street is no safer. (c) Over the long run, the probability that Main Street is found to be no safer for pedestrians when in reality fewer accidents are occurring is 0.05. (d) The probability of finding a p-value of 0.05 or larger assuming that Main Street is no safer is α. (e) The probability that, over the long run, the chicken uses the crosswalk to cross the road is 0.05.

(6) Now suppose the Pedestrian Safety Committee found a p-value of 0.0746. Which of the following statements is true? (a) The Committee should reject at $\alpha = 0.01$. (b) The Committee should reject at $\alpha = 0.05$ but not at $\alpha = 0.01$. (c) The Committee should reject at $\alpha = 0.08$ but not at $\alpha = 0.05$. (d) The Committee should reject at $\alpha = 0.10$ but not at $\alpha = 0.08$. (e) The Committee should not reject at $\alpha = 0.10$.

(7) Suppose we Fail to Reject H_0 for $\alpha = 0.05$. Then for $\alpha = 0.10$, we _____, (a) fail to reject H_0. (b) reject H_0. (c) not enough information.

(8) Suppose we Reject H_0 for $\alpha = 0.05$. Then for $\alpha = 0.10$, we _____. (a) fail to reject H_0. (b) reject H_0. (c) not enough information.

(9) Suppose we Fail to Reject H_0 for $\alpha = 0.05$. Then for $\alpha = 0.01$, we _____, (a) fail to reject H_0. (b) reject H_0. (c) not enough information..

(10) Suppose we Reject H_0 for $\alpha = 0.05$. Then for $\alpha = 0.01$, we _____. (a) fail to reject H_0. (b) reject H_0. (c) not enough information.

(11) A very small p-value means (a) the evidence for the alternative is small (i.e., the power is small). (b) the sample size is small. (c) the probability of rejecting the null is small (i.e., less likely to reject the null). (d) the evidence against the null is strong (more likely to reject the null). (e) None of the above.