

系所組別：土木工程學系甲、乙、丁組

考試科目：工程數學

考試日期：0223，節次：3

※ 考生請注意：本試題不可使用計算機

1. (20%) Find the general solution of following ODEs.

(a) (10%) $y' + \frac{1}{x}y = \cos(x)$

(b) (10%) $x^2y'' + 3xy' + y = \frac{2\ln(x)}{x}$

2. (20%) Find the eigenvalues and eigenfunctions of following Sturm-Liouville problem, and verify orthogonality.

$$(e^{2x}y')' + \lambda e^{2x}y = 0, \quad y(0) = 0, \quad y(\pi) = 0.$$

3. (20%) Using the Laplace transform, solve the following initial value problem.

$$y'' + 4y = \delta(t) - \delta(t - \pi), \quad y(0) = 0, \quad y'(0) = 0.$$

where $\delta(t)$ is the Dirac delta function.4. (20%) Let S be a piecewise smooth closed surface bounding a region V . Show that

$$\text{volume of } V = \frac{1}{3} \iint_S \mathbf{r} \cdot \mathbf{n} dS$$

where $\mathbf{r} = xi + yj + zk$.5. (20%) For a circular membrane subject to a pressure $p(x, y) = x$, the mathematic model is

$$\nabla^2 u = x, \quad u = 0 \text{ for } x^2 + y^2 = a.$$

Find the solution $u(x, y)$.