

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. A contractor is required by a city construction division to submit one, two, three, four, or five forms (depending on the nature of the project) for a building permit. Let  $Y$  = the number of forms required of the next applicant. The probability that the  $y$  forms are required is known to be proportion to  $y$  – that is,  $p(y) = ky$  for  $y = 1, \dots, 5$ . (1) What is the value of  $k$ ? (2) What is the probability that at most three forms are required? (3) What is the probability that between two and four forms (inclusive) are required? (4) Could  $p(y) = y^2/50$  for  $y = 1, \dots, 5$  be the pmf of  $Y$ ? (16 points)
2. The Young's modulus ( $E$ ) of samples of steel can be modeled by a normal distribution with a mean of 29,576 ksi and a standard deviation of 1,507 ksi. (12 points)
  - (1) What is the probability that a sample's  $E$  is between 28,000 and 29,500 ksi?
  - (2) If the 90<sup>th</sup> percentile value of the Young's modulus is desired, what's this  $E$  value?
  - (3) The commonly used Young's modulus for steel is 29,000 ksi. What percentile is this value located?

Values Provided for Your Calculations

$z$	-1.05	-0.38	-0.05	0.50	0.6	1.22	1.28	1.64	$t_{0.05, 6}$	$t_{0.025, 6}$	$t_{0.025, 16}$	$t_{0.025, 17}$
$\Phi(z)$	0.147	0.35	0.48	0.69	0.73	0.89	0.9	0.95	1.943	2.45	2.210	2.000

3. A study was conducted to analyze the derailment of a freight train due to the catastrophic failure of a traction motor armature bearing. A sample of 17 high-mileage traction motors was selected, and the amount of cone penetration (mm/10) was determined for pinion bearing and for the commutator armature bearing, resulting in the data shown below. (1) Calculate the 95% confidence interval for the population mean difference between penetration for the commutator armature bearing and penetration for the pinion bearing. (2) Would you say that the population mean difference has been precisely estimated? (3) Does it look as though population mean penetration differs for the two types of bearings? Explain. (12 points)

Variable	N	Mean	Median	StDev	Minimum	Maximum	Q1	Q3
Commutator	17	259.88	270	31.28	209	305	233	278
Pinion	17	264.06	273	27.41	208	315	243	281
Difference	17	-4.18	-4	35.85	-71	67	-19	1

4. Answer following four questions: (16 points)
  - (1) If two random variables  $X$  and  $Y$  have standard deviation of 7 and 24 respectively, what is the standard deviation of  $X-Y$ ?
  - (2) Suppose a certain type of fertilizer has an expected yield per acre of  $\mu_1$  with variance  $\sigma_2$ , whereas the expected yield for a second type of fertilizer is  $\mu_2$  with the same variance  $\sigma_2$ . Let  $S_1^2$  and  $S_2^2$  denote the sample variances of yields based on sample sizes  $n_1$  and  $n_2$ , respectively, of the two fertilizers. Show that the pooled (combined) estimator,  $\hat{\sigma}^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{n_1 + n_2 - 2}$ , is an unbiased estimator of  $\sigma^2$ .
  - (3) Suppose a researcher is interested in testing the hypotheses  $H_0: \mu = 6$  vs.  $H_a: \mu \neq 6$ . Her test statistic is 0.6. Calculate the p-value.

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(4) The following confidence intervals for  $\mu$  were found. 99%:(0.015, 0.447), 96%:(0.059, 0.404), 95%:(0.066, 0.396), and 90%:(0.093, 0.369). For testing the hypotheses  $H_0: \mu = 0.4$  vs.  $H_a: \mu \neq 0.4$ , what is the range for the p-value? Show your rationale. (a) 0.01 – 0.04 (b) 0.04 – 0.05 (c) 0.05 – 0.1 (d)  $< 0.01$  (e)  $> 0.1$

5. An experiment was carried out to compare electrical resistivity for six different low-permeability concrete bridge deck mixtures. There were 26 measurements on concrete cylinders for each mixture; these were obtained after 28-day curing. (1) Fill in the remaining ANOVA table shown below (5 points), (2) Write appropriate hypotheses (4 points), and (3) Make conclusion regarding the mean values (4 points)

Source	SS	df	MS	F	Table F = 2.27
Mixture					
Error		15			
Total	6000				

6. A professor of civil engineering wants to study the relationship between annual income ( $y$ , in \$1000s) and education ( $x$ , in years). A random sample of eight individuals is taken and the results are shown below: (31 points)

Parameter Estimates

Term	Estimate	Std Error	t-value	Prob> t
Intercept	10	4.35	2.44	0.0506
Education	3	0.35	8.44	0.0002

ANOVA

Source	SS	df	MS	F
Model	420			
Error				
Total	456			

- Fill in the blanks in the ANOVA table shown above.(7 points) (2) Calculate  $R^2$  and give an interpretation.(3 points)
- For an individual in the sample ( $x=16, y=59$ ), what is the residual based on the least squares estimate?(3 points)
- What is the probability that this individual ( $x=16$ ) will make at least \$61,000? (3 points)
- Is there any evidence to show that Education is useful to predict income? Set up appropriate hypotheses and carry out the test at a 0.05 level.(3 points)
- Based on the least squares estimate, if a person gets two years more of education, what would you expect the change in her income to be?(3 points)
- Is there statistically significant evidence to show that the slope is at least 2? Set up appropriate hypotheses and carry out the test at 0.05 level.(3 points)
- John has a two year college degree ( $x=14$ ). Calculate a 95% prediction interval for his income. ( $\bar{x}=12$ , stdev of  $x: s_x = 3$ )(3 points)
- Doug has less than 9 year education. The 95% prediction interval for Doug's income as compared with the 95% prediction interval for John's income will be wider \_\_\_\_\_ or narrower \_\_\_\_\_ or not enough information is given \_\_\_\_\_.(  $\bar{x}=12$  )(3 points)