系所組別：土木工程學系甲、乙、丁組

考試科目：工程數學　　　　　　　　　　　　　　　　考試日期：0211，節次：3

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※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (a) Find a 3×3 symmetric matrix **A** that has eigenvalues $\lambda_1 = 1, \lambda_2 = 3, \lambda_3 = 5$ and corresponding

    eigenvectors $\mathbf{X}_1 = \left\{\begin{matrix} 1 \\ -1 \\ 1 \end{matrix}\right\}, \mathbf{X}_2 = \left\{\begin{matrix} 1 \\ 0 \\ -1 \end{matrix}\right\}, \mathbf{X}_3 = \left\{\begin{matrix} 1 \\ 2 \\ 1 \end{matrix}\right\}.$

    (b) Evaluate $\mathbf{A}^3 - 9\mathbf{A}^2 + 23\mathbf{A} = ?$　　　　　　　　　　　　　　　　(20%)

2. The curve C is traced by the vector function $\mathbf{r}(t) = a\cos t\mathbf{i} + a\sin t\mathbf{j} + ct\mathbf{k}, 0 \le t \le 2\pi$, where a and c

    are constants. Find the

    (a) unit tangential and normal vector of C at $t = 0$,

    (b) curvature of C at $t = 0$,

    (c) arc length of C.　　　　　　　　　　　　　　　　(20%)

3. (a) Find the Fourier series expansion of the function $f(x) = x^2, 0 \le x < 2\pi$, and $f(x + 2\pi) = f(x)$.

    (b) Evaluate the series $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots$　　　　　　(20%)

4. Use the Laplace transform to solve the following problem.

    GE: $\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}, 0 < x < 1, t > 0$,

    I C: $u(x,0) = 0, \left.\dfrac{\partial u}{\partial t}\right|_{t=0} = \sin\pi x, 0 < x < 1$,

    BC: $u(0,t) = 0, u(1,t) = 0, t > 0$.　　　　　　　　　　　　(20%)

5. Evaluate

    (a) $\displaystyle\int_0^\infty \frac{dx}{1+x^4}$,

    (b) $\displaystyle\int_0^{2\pi} \frac{d\theta}{(2+\cos\theta)^2}$.　　　　　　　　　　　　　　　(20%)