※ 考生請注意：本試題可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Answer the following questions.

   (1) Time headway in traffic flow is the elapsed time between two consecutive cars. Let X = time headway between two randomly chosen cars. Consider the probability density function of X as follows.

$$f(x)=\begin{cases}0.15\bullet e^{-0.15(x-0.5)} & x\ge 0.5\\ 0 & otherwise\end{cases}$$

   (1.1.a.) Show that f(x) is a legitimate function. (5 points)　　(1.1.b.) $P(X\le 5)$ = ? (5 points)

   (2) Check the independence of the random variables X and Y. If the joint density of X and Y is as follows:

   (1.2.a.) $f(x, y) = x + y$, $0 < x < 1$, $0 < y < 1$. (5 points)　　(1.2.b.) $f(x, y) = 2e^{-x-2y}$, $0 < x, y < \infty$. (5 points)

   (3) Let X be a discrete random variable with probability mass function shown below. What is the variance of X? (5 points)

x	*-2*	*-1*	*1*	*2*
f(x)	0.4	0.2	0.2	0.2

2. Answer the following questions.

   (1) An instructor gives a test to a class containing several hundred students. It is known that the standard deviation of the scores is 14 points. A random sample of 49 scores is obtained. What is the probability that the average score of the students in the sample will differ from the overall average by more than 2 points? (5 points)

   (2) The breakdown strength of a randomly chosen material is known to be normally distributed with mean value of 40 MPa and standard deviation of 1.5 MPa. (2.2.a.) What is the probability that the breakdown strength of a single material is less than 41? (5 points) (2.2.b.) What value is such that only 15% of materials have breakdown strength exceeding that value? (5 points)

   (3) What is the confidence level for the $\left(\bar{x}-1.345\frac{s}{\sqrt{15}},\bar{x}+2.624\frac{s}{\sqrt{15}}\right)$? (5 points)

   (4) I want to know if the true proportion of Taiwan citizens who are over 75 is 5% or not. All I have is these 3 confidence intervals. 90%:(0.0211, 0.0589), 95%:(0.0174, 0.0626), and 99%:(0.0104, 0.0696). State the null and alternative hypotheses. Would you be able to reject or fail to reject the null hypothesis? Why? (5 points)

Values Provided for Your Calculations

z	-1	-0.38	-0.05	0.67	1	1.04	1.96	2.57	$t_{0.1,14}$	$t_{0.05,6}$	$t_{0.025,60}$	$t_{0.025,6}$	$t_{0.01,14}$
Φ(z)	0.16	0.35	0.48	0.75	0.84	0.85	0.975	0.995	1.345	1.943	2	2.447	2.624

3. Answer the following questions.

   (1) A researcher conducted an experiment in which the effects of an additive on strength of steel were studied. The strength under consideration was the steel at which steel deprived of heat presses a lever to obtain strength. Four dosage levels of the additive were studied, including a zero level. All dosage levels were specified in terms of milligrams of additive per kilogram of weight of the steel. For each dosage, sixteen steel bars of the same

type were used in a random order. The response variable was defined as the strength (in GPa) for the given treatment. The total sum of squares is 156 and the error sum of squares is 120 in this case.

Source	SS	df	MS	F
Between	(請勿在此作答)			
Within				
Total				

(3.1.a.) Finish the ANOVA table above. (5 points)

(3.1.b.) Conduct a test for equality of factor level means. Control the risk at 0.05. State the null and alternative hypothesis, and conclusion. Note Table F=2.76. (5 points)

(3.1.c.) $\overline{Y}_{1\bullet} = 1.8190$, $\overline{Y}_{2\bullet} = 1.8029$, $\overline{Y}_{3\bullet} = 1.6085$, $\overline{Y}_{4\bullet} = 0.9171$. Obtain a 95% confidence interval for $L = \frac{\mu_1 + \mu_2}{2} - \frac{\mu_3 + \mu_4}{2}$. Interpret your interval estimate. (5 points)

(2) A manufacturer of electric bulbs has a large warehouse filled with newly manufactured bulbs. Let μ and σ^2 denote the mean and variance, respectively, of the population of the lifetimes (in hundreds of hours) of all 75-watt bulbs in the warehouse. From previous studies, a value of 7.29 is available for σ^2.

(3.2.a.) What sample size should be used if it was desired to estimate to be within 100 hours with 99% confidence? (Assume that the lifetimes are normally distributed.) (5 points)

(3.2.b) Suppose that a quality control engineer selected 100 bulbs at random from the warehouse and measured the lifetime of each (in hundreds of hours) and computed the sample mean to be 11.51 hours. Compute the 95% confidence interval for μ. Explain what you mean by saying that you have 95% confidence in this interval (5 points)

4. An observational study was conducted to determine how well the number of calories in common food items (for example: Big Mac from MacDonald, Burger from Burger King, etc.) can predict the fat content in the food item. The following data were obtained on 8 food items.

$\Sigma X = 8{,}000$, $\Sigma Y = 400$, $\Sigma X^2 = 8{,}800{,}000$, $\Sigma XY = 432{,}000$, $S_{XX} = 8{,}000$, SST = 1,440, SSR = 360

(1) Estimate the slope and intercept from a simple linear regression analysis of these data. Note that

$$b_1 = \frac{\sum XY - [(\sum X \cdot \sum Y)/n]}{\sum X^2 - n \cdot (\sum X/n)^2} \quad \text{(5 points)}$$

(2) Finish the ANOVA table for the regression. Calculate the mean square of error. (5 points)

(3) State the null and alternative hypotheses for the ANOVA table. Use the ANOVA table to test these hypotheses using $\alpha = 0.05$. Note that Table F = 5.99 (5 points)

(4) The investigator on this study had hypothesized that the slope of the relationship relating fat to calories was .075 gm/calorie. Compute the estimated standard error of $\hat{\beta}_1$. Test this hypothesis versus the two-sided alternative. Interpret your findings using $\alpha = 0.05$. (5 points)

(5) Use the ANOVA table to calculate the R^2 value. Use this value to comment on how well the linear regression model fits the data. (5 points)