編號： 107

國立成功大學 105 學年度碩士班招生考試試題

系　所：土木工程學系

考試科目：工程數學

考試日期：0227，節次：3

※ 考生請注意：本試題不可使用計算機。　請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Given the ordinary differential equation,

$$\frac{d^2y}{dx^2}+y-2\sin x=0 \quad \text{with} \quad y(0)=0 \quad \text{and} \quad \left.\frac{dy}{dx}\right|_{x=0}=0.$$

(a) Find the homogeneous solutions and the particular solution at first, and then get the complete solution. (15%)

(b) Solve the ordinary differential equation by using the Laplace transform. (15%)

2. Many vectors, $\mathbf{x}_j,\ j=1,2,\ldots,n$, could be transformed by two given matrices of order n, $\mathbf{A}$ and $\mathbf{B}$, respectively, in order to produce two parallel vectors $\mathbf{Ax}_j$ and $\mathbf{Bx}_j$, where $\mathbf{A}\neq c\mathbf{B}$ and c is a constant.

(a) Under what assumptions on $\mathbf{A}$ and $\mathbf{B}$ is the following orthogonal condition satisfied?
$\mathbf{x}_j^T\mathbf{Bx}_k=0$ for any two non-parallel original vectors, $\mathbf{x}_j$ and $\mathbf{x}_k$. (5%)

(b) Prove the above orthogonal condition. (15%)

(c) If a vector is linearly expressed by those orthogonal vectors with respect to $\mathbf{B}$, i.e.,
$\mathbf{y}=a_1\mathbf{x}_1+a_2\mathbf{x}_2+\ldots+a_n\mathbf{x}_n$
find a formula to obtain the coefficient a_j. (5%)

3. Solve the Laplace equation,

$$\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}=0 \quad 0\le x\le\pi,\ 0\le y\le\pi$$

with the boundary conditions,

$$\begin{cases} u(x,0)=0,\ u(x,\pi)=x,\ 0\le x\le\pi \\ u(0,y)=0,\ u(\pi,y)=y,\ 0\le y\le\pi \end{cases}.$$

(30%)

4. Use the residue theorem to evaluate the integral,

$$\int_{-\infty}^{\infty}\frac{1}{x^4-14x^2+81}dx.$$

(15%)