

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Answer the following questions. (5 points each)

(1) Let X be a discrete random variable with a probability mass function shown below. Find (1.1.a) $E(X)$, (1.1.b) $Var(X)$, (1.1.c) $Var(5-2X)$ (1.1.d) $E[(5 + 2X)^2]$

x	0	1	2	3	4
P(x)	0.1	0.15	0.45	0.25	0.05

(2) Suppose X is a random variable with a probability density function (pdf) given below. (1.2.a) Find the value of k that makes this a legitimate pdf. (1.2.b) Find the cumulative density function (cdf) of X

$$f(x) = \begin{cases} kx^2(1-x), & 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

2. The hardness of a metal is determined by measuring its penetration depth into a testing surface using a device. Suppose that the hardness is normally distributed with mean 70 and standard deviation 3. (5 points each)

- (1) If a specimen is considered to be acceptable only if the hardness is between 73 and 75, what is the probability that a randomly chosen specimen has an acceptable hardness?
- (2) If the acceptable range is as in part (1) and the hardness of each of 100 randomly selected specimens is independently determined, what is the expected number of acceptable specimens among the 100?
- (3) What is the 85th percentile of the hardness values?
- (4) If the acceptable range of hardness is $(70 - 2c, 70 + 2c)$, for what value of c would 95% of all specimens have acceptable hardness?

Values Provided for Your Calculations

z	-1	-0.38	-0.05	0.67	1.04	1.64	1.96	2	$t_{0.05, 6}$	$t_{0.05, 5}$	$t_{0.05, 18}$	$t_{0.05, 17}$	$t_{0.05, 16}$
$\Phi(z)$	0.16	0.35	0.48	0.75	0.85	0.95	0.975	0.977	1.943	2.015	1.734	1.740	1.746

3. A researcher investigated two types of additive, i.e., A and B, for their improvement on the compressive strength of concrete samples. She presented the following summary data. Assume that the population of compressive strength of concrete samples is normally distributed. (5 points each).

Type	Sample size	Sample mean (MPa)	Sample SD (MPa)
A	6	12,200	420
B	12	12,000	360

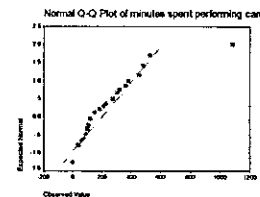
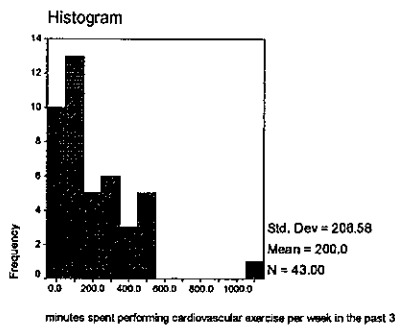
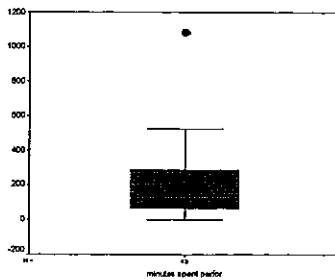
(1) Construct a 90% confidence interval for the overall average strength of concrete samples A.

- (2) Explain what you mean by saying that you have 90% confidence in this interval as in part (1).
- (3) What is the minimum sample size required if we wish to estimate the overall average strength of concrete samples A to be within 100 MPa with 90% confidence, if the overall standard deviation of the weights is 400 MPa?
- (4) Carry out a test of hypothesis at $\alpha = 0.05$ to decide whether the true compressive strength is larger among samples A than it is among samples B.
4. According to some experts, it is recommended that a person get 150 minutes of moderate-intensity activity per week. To determine whether or not NCKU students are getting enough exercise, a group of students took simple random selection (SRS) of size 43 from the NCKU phone book, and asked each student how many minutes they spent performing cardiovascular exercise per week in the past 3 months. The respondents did not have contact with one another. In this problem, you will be testing the claim that NCKU students got more than 150 minutes of cardiovascular exercise per week in the months of Feb., March, and April of 2016. Use the computer output below to test this claim. (5 points each)

Descriptives		Statistic
minutes spent performing cardiovascular exercise per week in the past 3 months.	Mean	200.00
	Median	120.00
	Std. Deviation	206.58

Binomial Test					
	Category	N	Observed Prop.	Test Prop.	Asymp. Sig. (2-tailed) ^a
minutes spent performing cardiovascular exercise per week in the past 3 months.	Group 1 ≤ 150	24	.56	.50	.542 ^a
	Group 2 > 150	19	.44		
Total		43	1.00		

a. Based on Z Approximation.



- What is the population in your claim? What is the variable of interest?
- If you were to do a t test of the claim that NCKU students got more than 150 minutes of cardiovascular exercise per week in the months of Feb., March, and April of 2016, what would the null and alternative hypotheses be?
- What is the p -value of your test? Explain your answer.
- If you are testing at the $\alpha = 0.05$ level, will you reject H_0 or will you fail to reject H_0 ? Explain your answer.
- State your conclusion in terms of the problem.
- Suppose the median amount of cardiovascular exercise NCKU students got in Feb., March, and April of 2016 were 170 minutes per week. Based on your conclusion above, will you have made a type I error, a type II error, or no error at all? Explain your answer.