

※ 考生請注意：本試題不可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. (20%) Find a general solution of the given ODE.

$$y' = xy / (1 - x^2)$$

2. (20%) Solve the IVP.

$$y'' - 4y = \sinh(2t), \quad y(0) = 0, \quad y'(0) = 1$$

3. (20%) Using the Laplace transforms to solve the IVP.

$$y'' + y = \delta(t) + u(t - \pi), \quad y(0) = 0, \quad y'(0) = 0.$$

4. (20%) The small free vertical vibration of a uniform elastic beam are modeled by the PDE

$$\frac{\partial^2 u}{\partial t^2} = -\frac{EI}{\rho A} \frac{\partial^4 u}{\partial x^4}.$$

If the beam is simply supported with the boundary conditions

$$\begin{aligned} u(0, t) = 0, & \quad u(L, t) = 0, \\ u_{xx}(0, t) = 0, & \quad u_{xx}(L, t) = 0, \end{aligned}$$

as well as the initial conditions

$$u(x, 0) = \sin(\pi x / L), \quad u_t(x, 0) = 0.$$

Find the solution $u(x, t)$.

5. (20%) Evaluate the following integral. Show details.

$$I = \int_{-\infty}^{\infty} \frac{dx}{x^4 - a^4}$$