

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Use the Laplace transform to solve the initial-value problem,

$$y'' + y = 3\sin 2t \text{ with } y(0) = 1 \text{ and } y'(0) = 1. \quad (20\%)$$

2. Use the Laplace transform and the results of Problem 1 to solve the boundary-value problem,

$$y'' + y = 3\sin 2t \text{ with } y(0) = 1 \text{ and } y(\pi/2) = 1. \quad (10\%)$$

3. Use the power series method to solve the initial-value problem,

$$y'' - 2xy' + 8y = 0 \text{ with } y(0) = 3 \text{ and } y'(0) = 0. \quad (25\%)$$

4. Verify Green's theorem by evaluating both integrals in $\oint_C (-y^2) dx + x^2 dy = \iint_R (2x + 2y) dA$,

$$\text{where } C \text{ is the circle } x^2 + y^2 = 9. \quad (20\%)$$

5. Solve the partial differential equation, $a^2 \frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 u}{\partial t^2}$, $0 \leq x \leq 1$, $t > 0$, where a is a constant,

$$\text{subject to the given conditions: } \begin{cases} u(0, t) = 0, u(1, t) = 0, t > 0 \\ u(x, 0) = x(1-x), \left. \frac{\partial u}{\partial t} \right|_{t=0} = x(1-x), 0 \leq x \leq 1 \end{cases} \quad (25\%)$$