

國立成功大學  
110學年度碩士班招生考試試題

編 號：102

系 所：土木工程學系

科 目：工程數學

日 期：0202

節 次：第 3 節

備 註：可使用計算機

※ 考生請注意：本試題可使用計算機。請於答案卷(卡)作答，於本試題紙上作答者，不予計分。

1. Solve the initial-value problem,  $(x+1)y' + y = \ln x$  with  $y(1) = -1$ . (10%)
2. Can the initial-value problem,  $x^2y'' - 3xy' + 4y = 0$  with  $y(1) = 5$  and  $y'(1) = 3$ , be solved by using the Laplace transform? And why? The solution is not necessary. (10%)
3. Find the general solution of the ordinary differential equation,  $xy'' - xy' + y = 0$ . (20%)
4. Evaluate  $e^{3A}$  if  $A = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ . (10%)
5. Verify Stokes' theorem by evaluating both integrals in  $\oint_C \mathbf{F} \cdot d\mathbf{r} = \iint_S (\text{curl } \mathbf{F}) \cdot \mathbf{n} dS$ ,  
where  $\mathbf{F} = y\mathbf{i} + (y-x)\mathbf{j} + z^2\mathbf{k}$ , and  $S$  is the portion of the sphere  $x^2 + y^2 + (z-4)^2 = 25$ ,  $z \geq 4$   
and is oriented upward. (20%)
6. Describe the procedures, briefly but clearly, to solve the following different partial differential equations subject to the given conditions. Solutions are not necessary.
  - (a)  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ ,  $0 \leq x \leq \pi$ ,  $t > 0$ ;  $\begin{cases} u(0, t) = 0, u(\pi, t) = 0, t > 0 \\ u(x, 0) = f(x), 0 \leq x \leq \pi \end{cases}$  (10%)
  - (b)  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ ,  $x \geq 0$ ,  $t > 0$ ;  $\begin{cases} u(0, t) = 0, t > 0 \\ u(x, 0) = f(x), x \geq 0 \end{cases}$  (10%)
  - (c)  $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t}$ ,  $-\infty < x < \infty$ ,  $t > 0$ ;  $u(x, 0) = f(x)$ ,  $-\infty < x < \infty$  (10%)