國立成功大學

114學年度碩士班招生考試試題

編　號：81

系　所：土木工程學系

科　目：工程數學

日　期：0210

節　次：第 3 節

注　意：1. 可使用計算機
　　　　2. 請於答案卷(卡)作答，於試題上作答，不予計分。

1. (15%) Consider the two-dimensional steady-state Poisson's equation:

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=\sin(xy),$$

in the region $0<x<\pi, 0<y<\pi$, subject to the following boundary conditions:

$$\begin{cases} u(0,y)=0, & 0\le y\le \pi \\ u(\pi,y)=0, & 0\le y\le \pi \\ u(x,0)=0, & 0\le x\le \pi \\ \dfrac{\partial u}{\partial y}(x,\pi)=0, & 0\le x\le \pi \end{cases}$$

Use separation of variables or another suitable method to find (or outline) an approximate form of the solution to this boundary-value problem (BVP).

2. (20%) Consider the second-order linear nonhomogeneous differential equation:

$$y''+4y'+5y=2e^{-2x}\cos(x)$$

(1) (12%) Find the general solution (including the homogeneous and particular solutions).

(2) (8%) Given the initial conditions $y(0)=1$ and $y'(0)=-1$, determine the corresponding particular solution.

Hint: Use the characteristic equation for the homogeneous part and an appropriate method (e.g., the method of undetermined coefficients or variation of parameters) for the nonhomogeneous part.

3. (15%) Given the matrix

$$\mathbf{A}=\begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

(1) (8%) Determine and prove whether $\mathbf{A}$ has only one eigenvalue $\lambda=2$ with algebraic multiplicity 3.

(2) (7%) Assuming $\lambda=2$ is indeed a triple root, find a complete set of linearly independent eigenvectors and generalized eigenvectors so that $\mathbf{A}$ can be brought into its Jordan normal form. Write down the Jordan form.

4. (25%) Define the piecewise function

$$f(x)=\begin{cases} x, & 0<x<1, \\ 2-x, & 1\le x<2, \\ 0, & \text{otherwise.} \end{cases}$$

(1) (13%) Find the Fourier series expansion of $f(x)$ over the interval $-2<x<2$.

(2) (12%) Obtain its Fourier integral (or Fourier transform) representation in the same context of periodic (or extended) definition.

5. (25%) Consider the following mixed initial-boundary value problem (IVP+BVP) for the wave equation:

$$\begin{cases} \dfrac{\partial^2 u}{\partial t^2}=c^2\dfrac{\partial^2 u}{\partial x^2}, & & 0<x<L, t>0, \\ u(0,t)=0, & u(L,t)=0, & t\ge 0, \\ u(x,0)=\sin\left(\dfrac{\pi x}{L}\right), & \dfrac{\partial u}{\partial t}(x,0)=x(L-x) & 0\le x\le L. \end{cases}$$

(1) (13%) Use separation of variables to find the general form of the solution for the wave equation.

(2) (12%) Given the "unusual" initial velocity $x(L-x)$, show how to employ a sine Fourier series expansion to satisfy the boundary and initial conditions. Briefly outline the key steps and write down the final solution.