國立成功大學

115學年度碩士班招生考試試題

編　號：74

系　所：土木工程學系

科　目：工程數學

日　期：0203

節　次：第 3 節

注　意：1.可使用計算機
2.請於答案卷(卡)作答，於試題上作答，不予計分。

1. (15%) Consider a simplified plane stress tensor represented by the matrix $\mathbf{A}=\begin{bmatrix}3 & 1 & 1\\ 1 & 3 & 1\\ 1 & 1 & 3\end{bmatrix}$.

   (1) (5%) Determine the characteristic equation and the corresponding eigenvalues of $\mathbf{A}$.

   (2) (10%) Find an orthogonal matrix $\mathbf{P}$ (where the column vectors of $\mathbf{P}$ are normalized eigenvectors) such that $\mathbf{P}^{\mathrm{T}}\mathbf{AP}=\mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix.

2. (15%) In the nonlinear analysis of a civil engineering structure, the equilibrium equation is given by $f(x)=x^3-2x-5=0$. Determine the root of this equation.

   (1) (5%) State the iterative formula for the Newton-Raphson method, expressing the next approximation x_{n+1} in terms of the current approximation x_n.

   (2) (10%) Assuming an initial guess of x_0=2, calculate the value of the first iteration x_1.

3. (20%) Solve the following second-order linear nonhomogeneous initial-value problem (IVP): $y''+2y'+5y=10\cos(x)$. The solution must satisfy the initial conditions: $y(0)=0$ and $y'(0)=0$.

4. (20%) Let V be the solid region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$. Let denote the entire boundary surface of V, oriented outward. A vector field is given by: $\mathbf{F}=x^2\mathbf{i}+y^2\mathbf{j}+(z+3)\mathbf{k}$

   (1) (5%) Calculate the divergence of $\mathbf{F}$.

   (2) (5%) Apply the Divergence Theorem to transform the surface flux integral $\iint_S \mathbf{F}\cdot\mathbf{n}\,dA$ into an equivalent volume integral.

   (3) (10%) Evaluate the integral to determine the total flux.

5. (15%) Consider the one-dimensional diffusion equation describing the dissipation of excess pore water pressure $u(z,t)$ in a soil layer: $\frac{\partial u}{\partial t}=c_v\frac{\partial^2 u}{\partial z^2}$, $0<z<H$, $t>0$. The system is subject to the boundary conditions $u(0,t)=0$ and $u(H,t)=0$. The initial condition is specified as $u(z,0)=u_0\sin(\frac{\pi z}{H})$. Obtain the solution $u(z,t)$ using the method of separation of variables or another appropriate analytical method.

6. (15%) A non-periodic piecewise function $f(x)$ is defined as: $f(x)=\begin{cases}2, & -1<x<1\\ 0, & \text{otherwise}\end{cases}$.

   (1) (10%) Derive the Fourier Integral Representation of $f(x)$. Explicitly state the expressions for the coefficients $A(\omega)$ and $B(\omega)$.

   (2) (5%) Determine the Fourier Transform of $f(x)$, denoted as $\hat{f}(\omega)=\mathcal{F}\{f(x)\}$.