1. 一彈性半平面 (elastic half plane), 受一集中力 P, 作用於原点 (如图(a)), 內部各点之位移函数, 求得为 $G(x, y)$

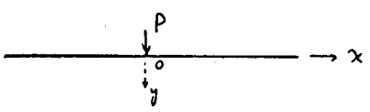

图 (a)

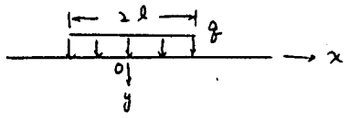

图 (b)

10% (a) 設一大小为 H 之集中力, 作用於 $x=3$, $y=0$ 處, 试決定其位移函数

10% (b) 试決定图 (b), (c) 情況下之位移函数

2. 常微分方程式 $xy'' + (1-x)y' + n'y = 0$ $(n = 0, 1, 2, \cdots)$

其解为 Laguarre polynomial $L_n(x)$ $(n = 0, 1, 2, \cdots)$

10% (a) 试將此方程式化为标準之 Sturm - Liouville 方程式形態

$$[r(x)y']' + [g(x) + \lambda p(x)]y = 0$$

決定出 $r(x)$, $g(x)$, λ, 与 $p(x)$

10% (b) 證明 $\int_0^\infty p(x) L_m(x) L_n(x)\, dx = 0$ $(m \neq n)$

並据以決定, 以 $\sum_{n=0}^{\infty} C_n L_n(x)$ 表示 $f(x)$, $x > 0$ 之係数 C_n

3. 一桿件若为 l, 各点溫度当 $t=0$ 時为 $f(x)$, 兩端溫度随时变化, 分别为 $\phi_0(t)$, $\phi_l(t)$, 設考慮傳導与幅射, 其控制方程式为:

$$k^2 \frac{\partial^2 \phi}{\partial x^2} - \lambda \phi = \frac{\partial \phi}{\partial t}, \quad \text{其中 } k, \lambda \text{ 为已知实数.}$$

10% (a) 令 $\phi = \psi e^{-\lambda t}$, 试列出以 $\psi(x,t)$ 表示之数学模式

10% (b) 若 $\phi_0(t) = 0$, $\phi_l(t) = l e^{-\lambda t}$. 令 $\psi = \psi_1 + \psi_2$, 可將上列数学模式分为兩個較簡易之子題, 试据以求解 $\phi(x,t)$.

4. 設 $f(x)$ $-1 \le x \le 1$ 为一連續函数, 且 $f(-1)$, $f(0)$, $f(1)$ 之值为已知

10% (a) 试以 Lagrangian 引式, 將 $f(x)$ 以一 2 階多項式近似表示之, 即 $f(x) \simeq l_0(x) f(-1) + l_1(x) f(0) + l_2(x) f(1)$

其中 $l_i(x)$ $(i = 0, 1, 2)$ 为 2 階多項式

10% (b) 由以上結果, 分别求 $\int_{-1}^{1} f(x)\, dx$, $f'(0)$ 之近似值

3. A bar of length L with weight W is initially pinned at end A and is connected to end C by a wire BC as shown in fig. A, if the bar is released from rest at $\theta = 60^{\circ}$ by cutting the wire BC, please determine the reaction at end A when the bar at the position of $\theta = 30^{\circ}$. (15%)

4. A ball A of mass 10 kg suspended from ceiling by a wire of length 1 m as shown in fig. B, is initially in horizontal position and released from rest to its lowest position to hit a block B of mass 15 kg which is connected to a spring with spring constant k = 90 N/m, whereas the coefficient of restitution between the ball and the block is e = 0.5. By assuming no frictions between all the contacting surfaces, please determine the maximum deformation of this spring. (15%)

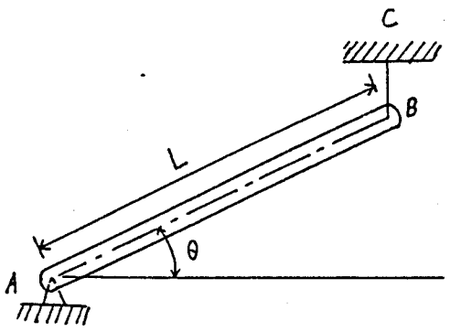

fig. A

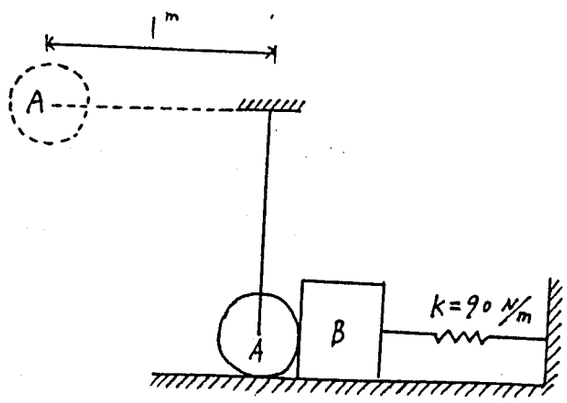

fig. B

5. A 180-N force is applied at corner B of a rigid piece of pipe ABDE as shown. The pipe is supported by the ball and socket joints A and E, respectively, and by a cable attached at the midpoint C of the portion BD and at a point G on the wall. Determine

1) the moment about point "A" due to the force W (5%)

2) the moment about axis AE due to the force W (10%)

3) the minimum tensile force (Tc) in the cable CG when the system is in equilibrium. (15%)