國立成功大學 78 學年度 土木研究所考試(工程數學 試題) 共 1 頁 第 1 頁

(丙組)

20% 1. Solve the ordinary differential equation

$$x y'' + y' - 4\frac{y}{x} = 1 \quad , \quad 1 \le x \le 2$$

$$y(1) = 0 \quad , \quad y(2) = 1$$

20% 2. A conservative vector field is given by

$$\vec{F} = (y^2 \cos x + z^3)\,\vec{i} + (2y \sin x - 4)\,\vec{j} + (3xz^2 + 2)\,\vec{k}$$

(a) Find the corresponding potential function ϕ, ($\vec{F} = \nabla\phi$).

(b) Evaluate the line integral $\int_c \vec{F}\cdot d\vec{r}$, where the path of integration is $x = 2t+1$, $y = t^2$, $z = -1$, $0 \le t \le 1$.

20% 3. The area element and volume element in Cartesian coordinates (x, y, z) are $dA = dx\,dy$, $dV = dx\,dy\,dz$. A transform of coordinates is given by $x = x(u, v, w)$, $y = y(u, v, w)$, $z = z(u, v, w)$.

(a) Derive the arc length ds in terms of du, dv, dw.

(b) Derive the expression for the area element in (u, v, w) coordinates

(c) Derive the expression for the volume element in (u, v, w) coordinates

20% 4. Denote the Laplace transform of $f(t)$ as $\mathcal{L}\{f(t)\} = F(s)$

(a) show that $F'(s) = \mathcal{L}\{-t\,f(t)\}$

(b) show that $\int_s^\infty F(x)\,dx = \mathcal{L}\{t^{-1} f(t)\}$

(c) $\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\} = ?$

20% 5. 設一交通流 (traffic flow) 中,車輛先後到達某一定点之時間間隔 (interarrival time) T 為指數分佈 (exponential distribution),且將 T 之或然率密度函數 (probability density function, PDF) 表示為 $f_T(t) = A\,e^{-Bt}$, $t \ge 0$, 其中 A, B 為大於零之常數.

(1) 依 PDF 之定義,证明 $A = B$

(2) 利用期望值 $E(T)$,說明常數 A 代表之意義.

(3) 求出 T 之累積分佈函數 (cumulation distribution function) $F_T(t)$.

(4) T 小於 $E(T)$ 之或然率為何?