(丙)

國立成功大學 79 學年度 土研所 考試(工程數學(丙)試題) 共 1 頁 第 1 頁

1. Bessel's equation is :

$$x^2y'' + xy' + (x^2-p^2)y = 0 \qquad (1)$$

(a) Determine the nature of the singularities at $x=0$ and at $x=1$.
(b) Determine the nature of the singularity at $x=\infty$ by transforming the independent variable to $t=1/x$.
(c) Determine the indicial equation of equation (1) (Do not solve it).

2. 1-dimensional Wave equation is

$$U_{tt} = U_{xx} \qquad -\infty < x < \infty, \quad t>0.$$

Find $U(x,2\pi)$ and $U(2\pi,t)$ if

$$U(x,0) = \begin{cases} \sin x & |x| \le \pi \\ 0 & |x| > \pi \end{cases}$$

$$U_t(x,0) = \begin{cases} 1 & |x| \le \pi \\ 0 & |x| > \pi \end{cases}.$$

3. Consider

$$\underset{\sim}{A} = \begin{bmatrix} 3 & 0 & -\sqrt{2} \\ 0 & 3 & 0 \\ -\sqrt{2} & 0 & 2 \end{bmatrix} \qquad \underset{\sim}{b} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

(a) Find the eigenvalues and corresponding eigenvectors of $\underset{\sim}{A}$.

(b) Solve $\underset{\sim}{A}\,\underset{\sim}{x} = \underset{\sim}{b}$ using Gaussian elimination and back substitution.

(c) Find the proper orthogonal matrix $\underset{\sim}{P}$ (i.e., $\underset{\sim}{P}^T = \underset{\sim}{P}^{-1}$) such that the solution to part b is given as $\underset{\sim}{x} = \underset{\sim}{P}\,\underset{\sim}{\Lambda}^{-1}\underset{\sim}{P}^T\underset{\sim}{b} = \underset{\sim}{A}^{-1}\underset{\sim}{b}$, where $\underset{\sim}{\Lambda}$ is the diagonal matrix whose elements are the eigenvalues of $\underset{\sim}{A}$.

4. (a) Solve completely : $(D-1)^2y = x^2e^x$ where $D = d/dx$.

(b) Find particular solution of $y'' + 2y' + 16y = \cos 2x$.

(c) Solve $y'' + \cos x\, y' + (1+\sin x)y = 0$ using the fact that $\cos x$ is a solution.

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5. Let $f = x^2 + y^2 + z^2$

$\underset{\sim}{u} = xz\underset{\sim}{i} + yz\underset{\sim}{k}$

Find (a) $\mathrm{div}(f\underset{\sim}{u})$

(b) $\nabla^2 f$

(c) $\mathrm{curl}(\mathrm{grad}\ f)$

(d) $\mathrm{div}(\mathrm{curl}\ \underset{\sim}{u})$

(e) $\mathrm{grad}(\underset{\sim}{u}\cdot\underset{\sim}{u})$