(乙)

1. Bessel's equation is :

$$x^2y'' + xy' + (x^2-p^2)y = 0 \qquad (1)$$

(a) Determine the nature of the singularities at $x=0$ and at $x=1$.
(b) Determine the nature of the singularity at $x=\infty$ by transforming the independent variable to $t=1/x$.
(c) Determine the indicial equation of equation (1) (Do not solve it).

2. 1-dimensional Wave equation is

$$U_{tt} = U_{xx} \qquad -\infty < x < \infty \quad , t>0 .$$

Find $U(x,2\pi)$ and $U(2\pi,t)$ if

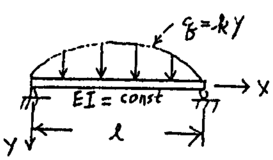
$$U(x,0) = \begin{cases} \sin x & |x| \le \pi \\ 0 & |x| > \pi \end{cases}$$

$$U_t(x,0) = \begin{cases} 1 & |x| \le \pi \\ 0 & |x| > \pi \end{cases}$$

3. A simple beam is subjected to a distributed load $q=ky$, where k is a positive constant and y is the lateral deflection. Using the deflection formula $EIy'''' = q$, show that the critical value of the load parameter k (At critical value, the beam will buckle) is determined by the equation:

$$\sin p\ell = 0$$

$$p = \sqrt[4]{k/EI}$$



4. (a) Solve completely : $(D-1)^2y = x^2e^x$ where $D = d/dx$.
(b) Find particular solution of $y'' + 2y' + 16y = \cos 2x$.
(c) Solve $y'' + \cos x\, y' + (1+\sin x)y = 0$ using the fact that $\cos x$ is a solution.

5. Let $f = x^2 + y^2 + z^2$

$\underset{\sim}{u} = xz\underset{\sim}{i} + yz\underset{\sim}{k}$

Find (a) $\mathrm{div}(f\underset{\sim}{u})$
(b) $\nabla^2 f$
(c) curl(grad f)
(d) $\mathrm{div}(\mathrm{curl}\ \underset{\sim}{u})$
(e) $\mathrm{grad}(\underset{\sim}{u}\cdot\underset{\sim}{u})$

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