1. Answer each of the following briefly.

(1) State the Fourier series of a function f(x). Your answer should include: property of f(x), Euler's formula for Fourier coefficient, and the convergent value at jump discontinuity. (5%)

(2) Classify the partial differential equations by the characteristic directions, and give an illustrated example for each case. (5%)

(3) For an analytic function $f(z) = u(x, y) + i\, v(x, y)$, $z = x + i\, y$, derive the Cauchy-Riemann equations. (5%)

2.

(1) Define eigenvalues and eigenvectors of a matrix **A**. (3%)

(2) Define positive definite and positive semi-definite for a general square matrix. (3%)

(3) If the matrix **A** is symmetric and positive definite, what can be said of its eigenvalues? (3%)

(4) Find the eigenvalues and eigenvectors of the matrices **A** and **B**. (8%)

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

(5) Find, if possible, orthogonal matrices **T** and **S** that diagonalize (not simultaneously) **A** and **B**. (i.e. $\hat{\mathbf{D}} = \mathbf{S}^{-1}\mathbf{A}\,\mathbf{S}$, and $\mathbf{D} = \mathbf{T}^{-1}\mathbf{B}\,\mathbf{T}$, $\hat{\mathbf{D}}$ and **D** diagonal) If this condition is not possible, explain why. (8%)

3. The equation for beam on elastic foundation is

$$EI\frac{d^4 v}{dx^4} + kv = q(x)$$

where v is the deflection of beam, E is Young's modulus, I is the moment of inertia, k is the modulus of the foundation, and q(x) is the applied load. Referring to Fig. 1,

(1) State the boundary conditions. (3%)

(2) Find the deflection of the beam. (12%)

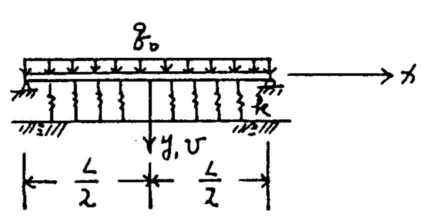

Fig. 1

4. A diffusion equation is given as

$$u_t = \alpha^2 u_{xx}$$

with the initial condition

$u(x, 0) = 100$, over $0 < x < L$

and the boundary conditions

$u(0, t) = 0$, $u(L, t) = 0$, for $0 < t < \infty$

Find $u(x, t)$. (15%)

5. Referring to Fig. 2, the wall of a harmonic oscillator is given some prescribed displacement $f(t)$. Assume that the system is initially at rest.

(1) Show that the equation of motion is

$$m\frac{d^2x}{dt^2} = k[f(t) - x] \qquad (3\%)$$

(2) Find the response $x(t)$ by the method of Laplace Transform for

(a) $f(t) = H(t-T)$, where $H(\ )$ is Heaviside step function, (6%)

(b) $f(t) = a\delta(t-t_0)$, $\delta(\ )$ is delta function. (6%)

6. Materials A and B can be mixed with the concrete to increase the hardness when the concrete is used in the construction of the pavement. The effects of such materials should be tested and compared with the pure concrete. The results of test are in terms of ratio of hardness. For material A, the ratios are 1.5, 1.4, 1.7, 1.5, and 1.4; while for material B, they are 1.2, 1.7, 1.9, 2.1, and 1.3.

(1) Estimate the mean and standard deviation of the effects for these two materials. (8%)

(2) Which one is better? Give your reason. (7%)

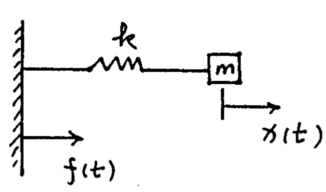

Fig. 2