1. Answer each of the following briefly.

(1) State the Fourier series of a function f(x). Your answer should include: property of f(x), Euler's formula for Fourier coefficient, and the convergent value at jump discontinuity. (5%)

(2) Classify the partial differential equations by the characteristic directions, and give an illustrated example for each case. (5%)

(3) For an analytic function $f(z) = u(x, y) + i\, v(x, y)$, $z = x + i\, y$, derive the Cauchy-Riemann equations. (5%)

2.

(1) Define eigenvalues and eigenvectors of a matrix **A**. (3%)

(2) Define positive definite and positive semi-definite for a general square matrix. (3%)

(3) If the matrix **A** is symmetric and positive definite, what can be said of its eigenvalues? (3%)

(4) Find the eigenvalues and eigenvectors of the matrices **A** and **B**. (8%)

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \qquad \mathbf{B} = \begin{bmatrix} 3 & 1 \\ 0 & 3 \end{bmatrix}$$

(5) Find, if possible, orthogonal matrices **T** and **S** that diagonalize (not simultaneously) **A** and **B**. (i.e. $\hat{\mathbf{D}} = \mathbf{S}^{-1}\mathbf{A}\,\mathbf{S}$, and $\mathbf{D} = \mathbf{T}^{-1}\mathbf{B}\,\mathbf{T}$, $\hat{\mathbf{D}}$ and **D** diagonal) If this condition is not possible, explain why. (8%)

3. The equation for beam on elastic foundation is

$$EI\frac{d^4 v}{dx^4} + kv = q(x)$$

where v is the deflection of beam, E is Young's modulus, I is the moment of inertia, k is the modulus of the foundation, and q(x) is the applied load. Referring to Fig. 1,

(1) State the boundary conditions. (3%) 104

(2) Find the deflection of the beam. (12%)

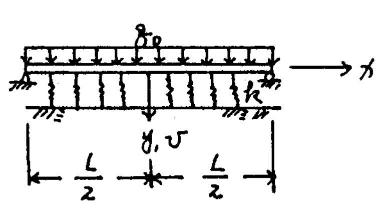

Fig. 1

4. A diffusion equation is given as

$$u_t = \alpha^2 u_{xx}$$

with the initial condition

$$u(x, 0) = 100, \text{ over } 0 < x < L$$

and the boundary conditions

$$u(0, t) = 0, \ u(L, t) = 0, \text{ for } 0 < t < \infty$$

Find u (x, t). (15%)

5. Referring to Fig. 2, the wall of a harmonic oscillator is given some prescribed displacement f(t). Assume that the system is initially at rest.

(1) Show that the equation of motion is

$$m\frac{d^2x}{dt^2} = k[f(t) - x] \quad (3\%)$$

(2) Find the response x(t) by the method of Laplace Transform for

(a) $f(t) = H(t-T)$, where H( ) is Heaviside step function, (6%)

(b) $f(t) = a\delta(t-t_0)$, $\delta($ $)$ is delta function. (6%)

6.

(1) State Residue Theorem. (5%)

(2) Evaluate the integral,

$$I = \int_0^\infty \frac{\cos ax}{x^2 + 4} dx, \quad (a \geq 0) \quad (10\%)$$

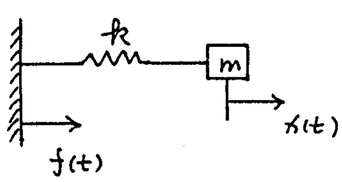

Fig. 2