國立成功大學 81 學年度土研所碩士班考試(工程數學甲組試題) 共 1 頁 第 1 頁

1. (18%) (i) Find the <u>particular</u> solution of the differential equation

$$x^2y'' - 3xy' + 4y = 2x + 4\ell n\, x.$$

(ii) Find the solution of the system of equations

$$\begin{cases} \dfrac{d^2x}{dt^2} + \dfrac{dx}{dt} + \dfrac{dy}{dt} + 2x + 2y = 0, \\[2ex] \dfrac{d^2x}{dt^2} + \dfrac{dy}{dt} + x + 2y = 0. \end{cases}$$

2. (14%) Find the eigenvalues and corresponding independent eigenvectors of the matrix A:

$$A = \begin{bmatrix} 5 & -3 & -2 \\ 8 & -5 & -4 \\ -4 & 3 & 3 \end{bmatrix}.$$

3. (18%) (i) Evaluate the line integral $\int_C \mathbf{F}\cdot d\mathbf{x}$, where C is a straight line from (1,2,3) to (4,6,0) and $\mathbf{F} = yz\,\mathbf{i} + xz\,\mathbf{j} + xy\,\mathbf{k}$.

(ii) Evaluate the surface integral $\iint_S (x\,\mathbf{i} + y\,\mathbf{j} + z\,\mathbf{k})\cdot\mathbf{n}\,d\sigma$, where S is the <u>lateral</u> surface of the finite cylinder $x^2 + y^2 = 4$, $0 < z < 8$, and $\mathbf{n}$ is the outward normal of the lateral surface. (<u>Note</u>, not including the top surface $z = 8$ and the bottom surface $z = 0$)

4. (16%) Use the residue theorem to evaluate the following integrals:

(i) $\displaystyle\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)^2}$, (ii) $\displaystyle\oint_C \frac{z^3 + 2z + 1}{z - 1}\,dz$, where C: $|z| = 2$.

5. (18%) Compute the solution $u(r,\theta)$ to the problem by separation of variables,

$$r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial \theta^2} = 0,\quad 0 < \theta < \pi/2,\quad 0 < r < 1,$$

$$u(r, 0) = \frac{\partial u}{\partial \theta}(r, \tfrac{\pi}{2}) = 0,\quad 0 < r < 1,$$

$$u(1, \theta) = \theta(\tfrac{\pi}{2} - \theta),\quad 0 < \theta < \pi/2.$$

6. (16%) The equation of spherical waves is

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) - \frac{\partial^2 u}{\partial t^2} = 0.$$

It is know that this equation can be simplified by introducing a new dependent variable $w = ru$. (i) Show that the equation for w is

$$\frac{\partial^2 w}{\partial r^2} - \frac{\partial^2 w}{\partial t^2} = 0,$$

(ii) and then write down the general solution of u.

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