

1. Solve the following systems of differential equations.

$$\begin{aligned} t \frac{dy}{dt} &= -4x + \ln t \\ t^2 \frac{d^2x}{dt^2} &= y + 4x \end{aligned} \quad (20)$$

2. Find the derivative of velocity field  $V(x, y, z) = x + 3y^2 + 4z^3$  at the point  $P : (0.5, 0.5, 2.0)$  in the normal direction of a surface  $z = 4x^2 + 4y^2$ .

(15)

3. Calculate the following integral

$$I = \int_C \vec{V} \cdot d\vec{r}, \quad (15)$$

from point  $P_1 : (0, \pi/2)$  to point  $P_2 : (1, 0)$ . Where  $\vec{V} = 3x^5 y^2 \vec{i} + x^6 y \vec{j}$ ,  $\vec{r} = x\vec{i} + y\vec{j}$ , and  $C$  denote a path  $x = \cos y$

4. " If  $A$  is an orthogonal matrix, and  $B$  is similar to  $A$ , then  $B$  is also an orthogonal one "

Is the above statement true? why?

(10)

5. Find the value of the following integral

$$I = \int_{-\infty}^{\infty} \frac{x e^{-ix}}{x^2 + ix + 2} dx, \quad (20)$$

where  $i$  denote the pure imaginary number  $(0, 1)$ .

6. Solve the following partial differential equation

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 1, \quad (20)$$

with boundary conditions  $\phi(0, y) = \phi(a, y) = \phi(x, 0) = \phi(x, b) = 0$ .