1. Solve the following systems of differential equations.

$$2\frac{dx}{dt} - 3y + \frac{dy}{dt} = 0$$
$$\frac{dx}{dt} + \frac{dy}{dt} = t \qquad (20)$$

2. Find the inverse $f(t)$ of

$$F(s) = \left\{\frac{s^3}{(s^2+\omega^2)^2}\right\}, \qquad (20)$$

3. Explain the following terminologies.
   a. Positive definite matrix
   b. Orthogonal matrix
   c. Hermitian matrix

(15)

4. Compute the following integral,

$$I = \int_C \vec{F} \bullet d\vec{r}, \qquad (20)$$

where $\vec{F} = 3x\vec{i} - 2\vec{j} + z\vec{k}$, $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, and C consists of the straight line segment from $(1,0,-2)$ to $(6,1,3)$ and then from $(6,1,3)$ to $(-1,-1,-4)$.

5. Solve the following partial differential equation.

$$c^2\frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = 0, \quad 0 < x < a, \quad 0 < t < \infty$$

with the boundary conditions

$$y(0,t) = y(a,t) = 0,$$

and initial conditions

$$y(x,0) = \sin^3 x, \quad \frac{\partial y}{\partial t}(x,0) = x\sin x, \qquad (25)$$

where a and c are contants