

1. Solve the following differential equation.

$$\frac{dy}{dx} + x \frac{d^2y}{dx^2} = \ln(x), \quad (20)$$

2. Solve the following integral equation,

$$y(t) = te^t - 2e^t \int_0^t e^{-\tau} y(\tau) d\tau, \quad (20)$$

3. Explain the following terminologies.

- a. Positive definite matrix
- b. Orthogonal matrix
- c. Hermitian matrix

(15)

4. Compute the following integral,

$$I = \int_C \vec{F} \cdot d\vec{r}, \quad (20)$$

where $\vec{F} = -3xy\vec{i} + 2y\vec{k}$, $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, C is the semicircle $x^2 + z^2 = 4$, $y = 1$, $z \geq 0$, oriented from $(2, 1, 0)$ to $(-2, 1, 0)$.

5. Solve the following partial differential equation.

$$c^2 \frac{\partial^2 y}{\partial x^2} - \frac{\partial^2 y}{\partial t^2} = 0, \quad 0 < x < a, \quad 0 < t < \infty$$

with the boundary conditions

$$y(0, t) = y(a, t) = 0,$$

and initial conditions

$$y(x, 0) = \sin^3 x, \quad \frac{\partial y}{\partial t}(x, 0) = x \sin x, \quad (25)$$

where a and c are constants