(1) Evaluate $\oint_c \frac{1}{z^3+8}dz$, where c is the circle of radius 1 about $2i$. (10%)

(2) Evaluate $\int_{-\infty}^{\infty} \frac{1}{x^6+64}dx$, ($x$ is real). (10%)

(3) Consider the transverse vibration problem of a homogeneous road with length π.

PDE: $\frac{\partial^4 u}{\partial x^4} = -\frac{1}{a^2}\frac{\partial^2 u}{\partial t^2}$ $(0<x<\pi,\ t>0)$

where $u(x, t)$ is the displacement, a is a constant.

(a) Let $u(x,t)=X(x)T(t)$ to obtain differential equations for X and T and the proper forms of X and T. (10%)

(b) Then determine $u(x,t)$ in the case of supported ends:

$$u(0,t)=u(\pi,t)=\frac{\partial^2 u}{\partial x^2}(0,t)=\frac{\partial^2 u}{\partial x^2}(\pi,t)=0 \quad (t>0)$$ (10%)

(4) Consider the mathematical model for the bending of a simply supported beam subject to a uniformly distributed load q shown in Fig. 4. The length, the bending rigidity and the flexural deflection of the beam is L, EI and y, respectively.

ODE: $EI\frac{d^4 y}{dx^4}=q$,

BC: $y(0)=y''(0)=0$,

$y(l)=y''(l)=0$.

Solve for $y(x)$ by using the Fourier sine half-range expansion method. (20%)

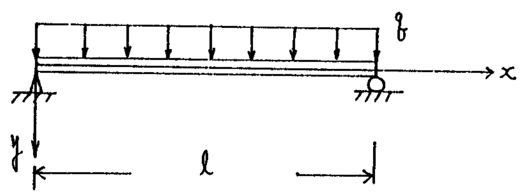

Fig. 4

(5) Consider the closed surface Σ consisting of the surface $z=\sqrt{x^2+y^2}$ for $x^2+y^2\le 1$ (Σ_1) and the flap cap Σ_2 consisting of the disk $x^2+y^2\le 1$ in the plane z=1 as shown in Fig. 5. An enclosed curve C represented as $x^2+y^2=1$ and z=1 is the boundary curve of Σ_2. A vector field **F** is given as $\mathbf{F}=-y\,\mathbf{i}+x\,\mathbf{j}-xyz\,\mathbf{k}$.

(a) Evaluate $\oint_c \mathbf{F}\cdot d\mathbf{r}$ (7%)

(b) Evaluate $\iint_{\Sigma_2} (\nabla \times \mathbf{F})\cdot \mathbf{n}\, d\sigma$ (7%)

(c) Is your answer of problem (5a) identical to that of problem (5b) ? (6%)
Justify your answer by stating the mathematical theorem used in the evaluation.

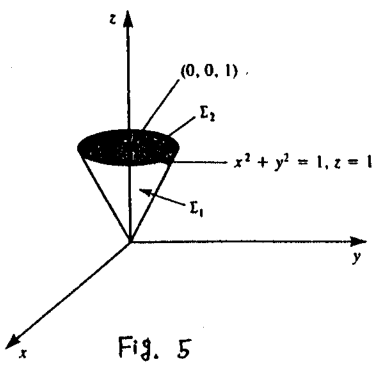

Fig. 5

(6) The equation of motion for the damped free vibration of a single-degree-of-freedom system (Fig. 6) is given by

$$m\frac{d^2x(t)}{dt^2}+c\frac{dx(t)}{dt}+k\,x(t)=0$$

(a) What is the position $x(t)$ measured from ? (5%)

(b) If the system is supplemented with the initial condition $x=x_0$, $\frac{dx}{dt}=v_0$ at t = 0, then find the general solution of $x(t)$ in the case of $c^2-4mk<0$. (10%)

(c) Plot your result for $x(t)$. (5%)

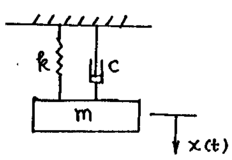

Fig. 6